W15 - Homework

Mean square error

01

🗹 MSE of a derived variable

Suppose that $X \sim \operatorname{Unif}([0,2])$, and $Y = X^2$.

You are using the sample mean with 50 samples, namely $M_{50}(Y)$, to estimate E[Y].

What is the mean square error of $M_{50}(Y)$? (Errors are deviations of this variable from E[Y].)

02

Z Estimates from joint PDF

Suppose *X* and *Y* have the following joint PDF:

$$f_{X,Y}(x,y) = egin{cases} rac{6(y-x)}{27} & 0 \leq x \leq y \leq 3 \ 0 & ext{otherwise} \end{cases}$$

(a) Find $f_X(x)$ and the blind estimate \hat{x}_B .

(b) Compute \hat{x}_G , the MMSE estimate of X assuming the event $G = \{X < 3/2\}$.

(c) Find $f_Y(y)$ and the blind estimate \hat{y}_B .

(d) Compute \hat{y}_H , the MMSE estimate of Y assuming the event $H = \{Y > 3/2\}$.

03

MMSE exact estimator from joint PDF

Suppose *X* and *Y* have the following joint PDF:

$$f_{X,Y}(x,y) = egin{cases} 2(y+x) & 0 \leq x \leq y \leq 1 \ 0 & ext{otherwise} \end{cases}$$

- (a) What is $\hat{x}_M(y)$, the MMSE estimate of X given Y = y?
- (b) What is $\hat{y}_M(x)$, the MMSE estimate of Y given X = x?

04

MMSE linear estimator from joint PDF

Suppose *X* and *Y* have the following joint PDF:

$$f_{X,Y}(x,y) = egin{cases} 2(y+x) & 0 \leq x \leq y \leq 1 \ 0 & ext{otherwise} \end{cases}$$

(a) What is $\hat{X}_L(Y)$, the MMSE linear estimator of X in terms of Y?

(b) What is $\hat{Y}_L(X)$, the MMSE linear estimator of Y in terms of X?

05

🗹 MMSE linear estimator from joint PMF

Suppose X and Y have the following joint PMF:

$Y\downarrow X ightarrow$	-1	0	1
1	$\frac{1}{6}$	$\frac{1}{12}$	0
3	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{24}$
5	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{1}{8}$
7	0	$\frac{1}{12}$	$\frac{1}{6}$

(a) Find the minimal MSE linear estimator for X in terms of Y.

(b) What is the MMSE error for this linear estimator?

(c) Use (a) to estimate X given Y = 1 and Y = 5.