W13 - Homework

Normal approximation - further problems

01

Normal approximation: Double ones

Roll a pair of dice 10,000 times. Estimate the odds that the number of "double ones" obtained is in the range $[290, 300]$.

02

Normal approximation: Heads v. tails

Flip a coin 10,000 times. Let H measure the number of heads, and T measure the number of tails. Estimate the probability that H and T are within 100 of each other.

Hint: Write an inequality for the condition, then sub a formula for T in terms of H.

Deviation and Large Numbers

03

Deviation estimation - Exponential

Let $X \sim \text{Exp}(\lambda)$ with $\lambda = 0.5$.

(a) Compute the Markov bound on $P[X > 5]$.

(b) Compute the Chebyshev bound on $P[X > 5]$.

(c) Find the exact value of $P[X > 5]$ and compare with yours answers in (a) and (b).

04

Deviation estimation - How many samples required?

Suppose the expected value of a score on the Probability final exam is 80 and the variance is 10. Assume the students' scores are independent.

How many students must take the exam before the average score in the class is known to lie within 5 points of 80 with a probability of 90%? What about 95%?

05

Deviation estimation - Factory production

Suppose a factory produces an average of 50 items per week.

(a) How likely is it that more than 75 items are produced this week? (Find an upper bound.)

(b) Suppose the variance is known to be 25 items. Now what can you say about (a)? (Hint: Monotonicity.)

(c) What do you know about the probability that the number of items produced differs from the average by at most 10?

06

★ Random walk forward

You play a game where you roll a die, and if the outcome is 1 or 2 you take a step forward, otherwise you take two steps forward. Let X_n be your position (measured in steps forward) after playing the game n times.

(a) Estimate $P[\, X_{90} \geq 160 \,]$ using a normal approximation for a certain relevant binomial distribution.

(b) Find $\lim_{n\to\infty} P[\, X_n > 1.6n \,]$ and $\lim_{n\to\infty} P[\, X_n > 1.7n \,]$.

Hint: Rewrite the conditions into a form where you can apply the Law of Large Numbers.

Significance testing

07

Testing paperclips - Likelihood of error

A factory assembly line machine is cutting paperclips to length before folding. Each paperclip is supposed to be 3 in long. The length of paperclips is approximately normally distributed with standard deviation 0.2 in.

(a) Design a significance test with $\alpha = 0.02$ that is based on the average of 5 measurements (sample mean). What is the rejection region? What is the probability of Type I error?

(b) What is the probability of Type II error, given that the average paperclip length on the machine is actually 3.1 in?