W08 - Homework

Functions on two random variables

01

Review practice: Function on one variable

Suppose the PDF of X is given by:

$$f_X(x) = \begin{cases} \frac{2}{3}x & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the CDF and PDF of $W = \ln X$.

02

PDF of min and max

Suppose $X \sim \text{Exp}(2)$ and $Y \sim \text{Exp}(3)$. Find:

- (a) The PDF of $W = \text{Max}(X, Y)$
- (b) The PDF of $W = \text{Min}(X, Y)$

Sums of random variables

03

PDF of sum from joint PDF

Suppose the joint PDF of X and Y is given by:

$$f_{X,Y} \quad = \quad \begin{cases} \frac{8}{81}xy & 0 \leq y \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the PDF of $X + Y$.

04

★ Poisson plus Bernoulli

Suppose that:

- $X \sim \text{Pois}(\lambda)$
- $Y \sim \text{Ber}(p)$
- Assume that X and Y are independent

Find a formula for the PMF of $X + Y$.

Apply your formula with $\lambda = 2$ and $p = 0.3$ to find $P_{X+Y}(7)$.

05

Convolution for uniform distributions over intervals

Suppose that:

- $X \sim \mathrm{Unif}[a, b]$
- $Y \sim \mathrm{Unif}[c, d]$
- X and Y are independent

Find the PDF of $X + Y$.

(You may find it helpful to start by considering specific numbers for a, b, c, d.)

06

★ Sums of normals

- (a) Suppose $X, Y \sim \mathcal{N}(\mu, \sigma^2)$ are independent variables. Find the values of μ and σ for which $X + X \sim X + Y$, or prove that none exist.
- (b) Suppose $\mu = 0$, $\sigma = 1$ in part (a). Find $P[X > Y + 2]$.
- (c) Suppose $X \sim \mathcal{N}(0, \sigma_X)$ and $Y \sim \mathcal{N}(0, \sigma_Y)$. Find $P[X - 3Y > 0]$.