W07 - Homework

Joint distributions

01

Finish a PMF table - Strange families

Suppose that 15 percent of the families in a strange community have no children, 20 percent have 1 child, 35 percent have 2 children, and 30 percent have 3 children. Assume the odds of a child being a boy or a girl are equal.

If a family is chosen at random from this community, then B, the number of boys, and G, the number of girls, in this family will have the joint PMF partially shown in Table 6.2:

Table 6.2 $P\{B = i, G = j\}$.

i \ j	0	1	2	3	Row sum $= P\{B = i\}$
0	.15	.10		.0375	.3750
1	.10	.175		0	.3875
2			0	0	.2000
3	.0375	0	0	0	.0375
Column sum $= P\{G = j\}$	.3750	.3875	.2000	.0375	

(a) Complete the table by finding the missing entries.

(b) What is the probability that "B or G is 1"?

02

PMF calculations from a table

Suppose the joint PMF of X and Y has values given in this table:

$X \backslash Y$	**0**	**1**	**2**	**3**
1	0.10	0.15	0	0.05
2	0.20	0.05	0.05	0.20
3	0.05	0	x	0.05

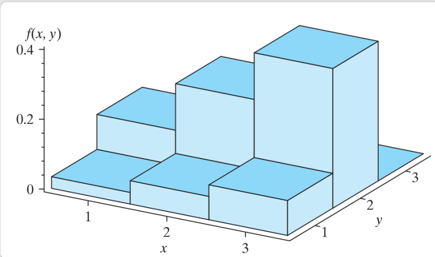
- (a) Find x.
- (b) Find the marginal PMF of X.
- (c) Find the PMF of the random variable $Z = XY$.
- (d) Find $P[X = Y]$ and $P[X > Y]$.

03

Marginals from joint PMF

Suppose the discrete joint PMF of X and Y is given by:

$$P_{X,Y}(x,y) \quad = \quad \frac{xy^2}{30}, \quad x = 1,2,3, \quad y = 1,2$$



Compute the marginal PMFs $P_X(x)$ and $P_Y(y)$.

04

Joint CDF on box events: All four corners

Consider the following formula:

$$P\big[x_1 < X \leq x_2,\, y_1 < Y \leq y_2\big] =$$

$$F_{X,Y}(x_2,y_2) - F_{X,Y}(x_2,y_1) - F_{X,Y}(x_1,y_2) + F_{X,Y}(x_1,y_1)$$

Prove this formula. Hint: Do these steps along the way:

- Draw these events in the xy-plane:

$$\begin{aligned} A &= \big\{X \leq x_1,\, y_1 < Y \leq y_2\big\} \\ B &= \big\{x_1 < X \leq x_2,\, Y \leq y_1\big\} \\ C &= \big\{x_1 < X \leq x_2,\, y_1 < Y \leq y_2\big\} \end{aligned}$$

- Draw the event $A \cup B \cup C$. Write the probability of this event in terms of $F_{X,Y}$.

05

Marginals from PDF

Suppose X and Y have joint PDF given by:

$$f_{X,Y}(x,y) = \begin{cases} 2e^{-(x+2y)} & \text{if } x,y > 0 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the marginal PDFs for X and Y.
- (b) Find $P[X > Y]$.

Independent random variables

06

Random point in a triangle

Consider a joint distribution whose PDF is constant inside the triangle with vertices $(0,0), (0,1)$, and $(1,0)$, and zero outside. Suppose a point (X,Y) is chosen at random according to this distribution.

- (a) Find the joint PDF $f_{X,Y}$.
- (b) Find the marginal PDFs for X and Y.
- (c) Are X and Y independent?

07

★ Factorizing the density

Consider two joint density functions for X and Y:

$$\begin{aligned} f_1(x,y) &= 6e^{-2x}e^{-3y}, \qquad x,y > 0, \\ f_2(x,y) &= 2yxe^{x^2}, \qquad x,y \in [0,1],\ x+y \in [0,1]. \end{aligned}$$

(Assume the densities are zero outside the given domain.)

Supposing f_1 is the joint density, are X and Y independent? Why or why not?
Supposing f_2 is the joint density, are X and Y independent? Why or why not?

08

★ Composite PDF from joint PDF

The joint density of random variables X and Y is given by:

$$f_{X,Y}(x,y) \quad = \quad \begin{cases} e^{-x-y} & x,y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Compute the PDF of X/Y. (Hint: First find the CDF of X/Y.)