

W06 - Homework

The symbol ★ indicates higher difficulty.

Normal distribution

01

✍ Converting to standard normal

Let $X \sim \mathcal{N}(\mu, \sigma^2)$. Show that $Z = \frac{X - \mu}{\sigma}$ is a standard normal random variable.

In other words, verify that the PDF of Z is $\varphi_Z(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$.

Hint: Use a method analogous to one in the lecture notes.

02

✍ Symmetry of φ

Show that $\Phi(-x) = 1 - \Phi(x)$ for all $x \in \mathbb{R}$.

03

✍ Generalized normal - misc

Let X be generalized normal variable with $\mu = 3$ and $\sigma = 2$. Using a chart of Φ values, find:

- (a) $P[2 < X < 6]$
- (b) c such that $F_X(c) = 0.67$
- (c) $E[X^2]$ (Hint: Use μ and σ to avoid integration.)

04

✍ Normal distribution - test scores

In a large probability theory exam, the scores are normally distributed with a mean of 75 and a standard deviation of 10.

- (a) What is the probability that a student scored between 70 and 80
- (b) What is the lowest score a student can achieve to be in the top 5%?
- (c) What score corresponds to the 25th percentile?

05

✍ Normal distribution - cars passing toll booth

The number of cars passing a toll booth on Wednesdays has a normal distribution $\mathcal{N}(1200, 40000)$.

- (a) What is the probability that on a randomly chosen Wednesday, more than 1,400 cars pass the toll booth?
- (b) What is the probability that between 1,000 and 1,400 cars pass the toll booth on a random Wednesday?

- (c) Suppose it is also known that at least 1200 cars passed the toll booth last Wednesday. What is the probability that in fact 1300 cars passed the toll booth that day?