W04 - Homework

In these problems, ★ indicates higher likely difficulty.

Bernoulli process

01

Binomial variable: guessing on a test

Your odds of getting any given exam question right are 0.80. The exam has 4 questions, and you need to answer 3 correctly to pass.

- (a) What is the probability that you pass?
- (b) Suppose that during the exam, you are 100% confident that you got the second question right. Now what are the odds that you pass?

02

PMF and CDF: number of heads in five flips

Let X count the number of heads resulting from five flips of a coin.

Write complete formulas (using cases) for the PMF and CDF.

03

Rolling until a six

A fair die is rolled until a six comes up.

What are the odds that it takes at least 10 rolls? Use a geometric random variable.

04

Intersection accidents

Suppose that the odds of an accident occurring on any given day at the intersection of Ivy and Emmet is 0.05.

What are the odds of the first accident occurring after day 4 and by day 10?

(You have calculated the answer before. This time, rework the problem in terms of an appropriate distribution type.)

05

★ Components of a car

A very strange car with n components will drive if at least half of its components work. Each component will work with the same probability p, independently of the others.

For what values of p is a car with $n = 3$ more likely to drive than a car with $n = 5$?

(Start by defining a random variable that counts the number of working components.)

06

Geometric distribution is memoryless

Suppose that $X \sim \text{Geom}(p)$.

Derive this equation:

$$P\big[X = n + k \;\big|\; X > n\big] = P[X = k]$$

Interpret the equation. (Inspired by the title.)

07

★ Binomial ratios

Suppose $X \sim \text{Bin}(n, p)$.

- Find the value of k that maximizes $P_X(k)$. Do this by studying the successive ratios $P_X(k)/P_X(k-1)$.
- Use these ratios to compute $P[X \leq 4]$ as a sum of 5 terms without using factorials. Do this by computing $P_X(0)$ directly, and then writing a recursive algorithm that determines $P_X(k)$ in terms of $P_X(k-1)$.

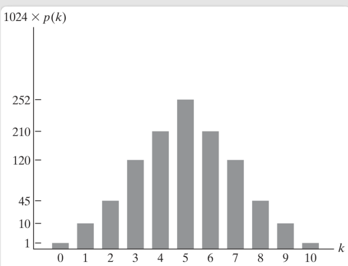

Figure 4.6 Graph of $p(k) = \binom{10}{k}\left(\frac{1}{2}\right)^{10}$.

08

Prize on the Mall

A booth on the Mall is running a secret prize game, in which the 5^{th} passerby wearing a hat wins $1,000.

Passersby wear hats independently of each other and with probability 20%.

Let N be a random variable counting how many passersby pass by before a winner is found.

- (a) What is the name of the distribution for N? What are the parameters?
- (b) What is the probability that the 10^{th} passerby wins the prize?
- (c) What is the probability that at least 7 passersby are needed before a winner is found?

Expectation and variance

09

★ Students and buses expect different crowding

Bus One has 10 students, Bus Two has 20, Bus Three has 30, and Bus Four has 40.

- Let X measure the number of students on a given random student's bus.
- Let Y measure the number of students on a given random driver's bus.

Compute $E[X]$ and $E[Y]$. Are they different? Why or why not?

10

Insurance expected payout

A car insurance analytics team estimates that the cost of repairs per accident is uniformly distributed between \$100 and \$1500. The manager wants to offer customers a policy that has a \$500 deductible and covers all costs above the deductible.

How much is the expected payout per accident?

(Hint: Graph the PDF for the cost of repairs X; write a formula for the payout in terms of X using cases; then integrate.)

11

★ Expectation, variance of geometric variable

Derive formulas for $E[X]$ and $\mathrm{Var}[X]$ given $X \sim \mathrm{Geom}(p)$.

Poisson process

[All on this topic are in HW for W05.]