DISCRETE RANDOM VARIABLES

Bernoulli (p) For $0 < p < 1$:

$$P_X(x) = \begin{cases} 1-p & x = 0 \\ p & x = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = p$$

$$\text{Var}[X] = p(1-p)$$

Binomial (n, p) For a positive integer n and $0 < p < 1$:

$$P_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x = 0, 1, 2, \dots, n \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = np$$

$$\text{Var}[X] = np(1-p)$$

Discrete Uniform (k, l) For integers k and l such that $k < l$:

$$P_X(x) = \begin{cases} \dfrac{1}{l-k+1} & x = k, k+1, \dots, l \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{k+l}{2}$$

$$\text{Var}[X] = \frac{(l-k)(l-k+2)}{12}$$

Geometric (p) For $0 < p < 1$:

$$P_X(x) = \begin{cases} p(1-p)^{x-1} & x = 1, 2, 3, \dots \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{1}{p}$$

$$F_X(x) = \begin{cases} 1-(1-p)^x & x = 1, 2, 3, \dots \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Var}[X] = \frac{1-p}{p^2}$$

Pascal (k, p) For positive integer k and $0 < p < 1$:

$$P_X(x) = \begin{cases} \binom{x-1}{k-1} p^k (1-p)^{x-k} & x = k, k+1, k+2, \dots \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{k}{p}$$

$$\text{Var}[X] = \frac{k(1-p)}{p^2}$$

Poisson (α) For $\alpha > 0$:

$$P_X(x) = \begin{cases} \dfrac{\alpha^x e^{-\alpha}}{x!} & x = 0, 1, 2, \dots \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \alpha$$

$$\text{Var}[X] = \alpha$$

CONTINUOUS RANDOM VARIABLES

Erlang (n, λ) For $\lambda > 0$ and a positive integer n:

$$f_X(x) = \begin{cases} \dfrac{\lambda^n x^{n-1} e^{-\lambda x}}{(n-1)!} & x \geq 0 \\ 0 & x < 0 \end{cases} \qquad E[X] = \frac{n}{\lambda}$$

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x} \displaystyle\sum_{k=0}^{n-1} \frac{(\lambda x)^k}{k!} & x \geq 0 \\ 0 & x < 0 \end{cases} \qquad \text{Var}[X] = \frac{n}{\lambda^2}$$

Exponential (λ) For $\lambda > 0$:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \qquad E[X] = \frac{1}{\lambda}$$

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \qquad \text{Var}[X] = \frac{1}{\lambda^2}$$

Gaussian (μ, σ) For $\sigma > 0$ and $-\infty < \mu < \infty$:

$$f_X(x) = \frac{e^{-(x-\mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}} \qquad E[X] = \mu$$

$$\text{Var}[X] = \sigma^2$$

Rayleigh (a) For $a > 0$:

$$f_X(x) = \begin{cases} a^2 x\, e^{-a^2 x^2/2} & x \geq 0 \\ 0 & x < 0 \end{cases} \qquad E[X] = \sqrt{\frac{\pi}{2a^2}}$$

$$F_X(x) = \begin{cases} 1 - e^{-a^2 x^2/2} & x \geq 0 \\ 0 & x < 0 \end{cases} \qquad \text{Var}[X] = \frac{4-\pi}{2a^2}$$

Uniform (a, b) For constants $a < b$:

$$f_X(x) = \begin{cases} \dfrac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases} \qquad E[X] = \frac{a+b}{2}$$

$$F_X(x) = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & a \leq x \leq b \\ 1 & x > b \end{cases} \qquad \text{Var}[X] = \frac{(b-a)^2}{12}$$

Standard Normal CDF $\Phi(z)$

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
0.00	0.5000	0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.97725	2.50	0.99379
0.01	0.5040	0.51	0.6950	1.01	0.8438	1.51	0.9345	2.01	0.97778	2.51	0.99396
0.02	0.5080	0.52	0.6985	1.02	0.8461	1.52	0.9357	2.02	0.97831	2.52	0.99413
0.03	0.5120	0.53	0.7019	1.03	0.8485	1.53	0.9370	2.03	0.97882	2.53	0.99430
0.04	0.5160	0.54	0.7054	1.04	0.8508	1.54	0.9382	2.04	0.97932	2.54	0.99446
0.05	0.5199	0.55	0.7088	1.05	0.8531	1.55	0.9394	2.05	0.97982	2.55	0.99461
0.06	0.5239	0.56	0.7123	1.06	0.8554	1.56	0.9406	2.06	0.98030	2.56	0.99477
0.07	0.5279	0.57	0.7157	1.07	0.8577	1.57	0.9418	2.07	0.98077	2.57	0.99492
0.08	0.5319	0.58	0.7190	1.08	0.8599	1.58	0.9429	2.08	0.98124	2.58	0.99506
0.09	0.5359	0.59	0.7224	1.09	0.8621	1.59	0.9441	2.09	0.98169	2.59	0.99520
0.10	0.5398	0.60	0.7257	1.10	0.8643	1.60	0.9452	2.10	0.98214	2.60	0.99534
0.11	0.5438	0.61	0.7291	1.11	0.8665	1.61	0.9463	2.11	0.98257	2.61	0.99547
0.12	0.5478	0.62	0.7324	1.12	0.8686	1.62	0.9474	2.12	0.98300	2.62	0.99560
0.13	0.5517	0.63	0.7357	1.13	0.8708	1.63	0.9484	2.13	0.98341	2.63	0.99573
0.14	0.5557	0.64	0.7389	1.14	0.8729	1.64	0.9495	2.14	0.98382	2.64	0.99585
0.15	0.5596	0.65	0.7422	1.15	0.8749	1.65	0.9505	2.15	0.98422	2.65	0.99598
0.16	0.5636	0.66	0.7454	1.16	0.8770	1.66	0.9515	2.16	0.98461	2.66	0.99609
0.17	0.5675	0.67	0.7486	1.17	0.8790	1.67	0.9525	2.17	0.98500	2.67	0.99621
0.18	0.5714	0.68	0.7517	1.18	0.8810	1.68	0.9535	2.18	0.98537	2.68	0.99632
0.19	0.5753	0.69	0.7549	1.19	0.8830	1.69	0.9545	2.19	0.98574	2.69	0.99643
0.20	0.5793	0.70	0.7580	1.20	0.8849	1.70	0.9554	2.20	0.98610	2.70	0.99653
0.21	0.5832	0.71	0.7611	1.21	0.8869	1.71	0.9564	2.21	0.98645	2.71	0.99664
0.22	0.5871	0.72	0.7642	1.22	0.8888	1.72	0.9573	2.22	0.98679	2.72	0.99674
0.23	0.5910	0.73	0.7673	1.23	0.8907	1.73	0.9582	2.23	0.98713	2.73	0.99683
0.24	0.5948	0.74	0.7704	1.24	0.8925	1.74	0.9591	2.24	0.98745	2.74	0.99693
0.25	0.5987	0.75	0.7734	1.25	0.8944	1.75	0.9599	2.25	0.98778	2.75	0.99702
0.26	0.6026	0.76	0.7764	1.26	0.8962	1.76	0.9608	2.26	0.98809	2.76	0.99711
0.27	0.6064	0.77	0.7794	1.27	0.8980	1.77	0.9616	2.27	0.98840	2.77	0.99720
0.28	0.6103	0.78	0.7823	1.28	0.8997	1.78	0.9625	2.28	0.98870	2.78	0.99728
0.29	0.6141	0.79	0.7852	1.29	0.9015	1.79	0.9633	2.29	0.98899	2.79	0.99736
0.30	0.6179	0.80	0.7881	1.30	0.9032	1.80	0.9641	2.30	0.98928	2.80	0.99744
0.31	0.6217	0.81	0.7910	1.31	0.9049	1.81	0.9649	2.31	0.98956	2.81	0.99752
0.32	0.6255	0.82	0.7939	1.32	0.9066	1.82	0.9656	2.32	0.98983	2.82	0.99760
0.33	0.6293	0.83	0.7967	1.33	0.9082	1.83	0.9664	2.33	0.99010	2.83	0.99767
0.34	0.6331	0.84	0.7995	1.34	0.9099	1.84	0.9671	2.34	0.99036	2.84	0.99774
0.35	0.6368	0.85	0.8023	1.35	0.9115	1.85	0.9678	2.35	0.99061	2.85	0.99781
0.36	0.6406	0.86	0.8051	1.36	0.9131	1.86	0.9686	2.36	0.99086	2.86	0.99788
0.37	0.6443	0.87	0.8078	1.37	0.9147	1.87	0.9693	2.37	0.99111	2.87	0.99795
0.38	0.6480	0.88	0.8106	1.38	0.9162	1.88	0.9699	2.38	0.99134	2.88	0.99801
0.39	0.6517	0.89	0.8133	1.39	0.9177	1.89	0.9706	2.39	0.99158	2.89	0.99807
0.40	0.6554	0.90	0.8159	1.40	0.9192	1.90	0.9713	2.40	0.99180	2.90	0.99813
0.41	0.6591	0.91	0.8186	1.41	0.9207	1.91	0.9719	2.41	0.99202	2.91	0.99819
0.42	0.6628	0.92	0.8212	1.42	0.9222	1.92	0.9726	2.42	0.99224	2.92	0.99825
0.43	0.6664	0.93	0.8238	1.43	0.9236	1.93	0.9732	2.43	0.99245	2.93	0.99831
0.44	0.6700	0.94	0.8264	1.44	0.9251	1.94	0.9738	2.44	0.99266	2.94	0.99836
0.45	0.6736	0.95	0.8289	1.45	0.9265	1.95	0.9744	2.45	0.99286	2.95	0.99841
0.46	0.6772	0.96	0.8315	1.46	0.9279	1.96	0.9750	2.46	0.99305	2.96	0.99846
0.47	0.6808	0.97	0.8340	1.47	0.9292	1.97	0.9756	2.47	0.99324	2.97	0.99851
0.48	0.6844	0.98	0.8365	1.48	0.9306	1.98	0.9761	2.48	0.99343	2.98	0.99856
0.49	0.6879	0.99	0.8389	1.49	0.9319	1.99	0.9767	2.49	0.99361	2.99	0.99861

Complementary CDF $Q(z)$

z	$Q(z)$	z	$Q(z)$	z	$Q(z)$	z	$Q(z)$	z	$Q(z)$
3.00	$1.35 \cdot 10^{-3}$	3.40	$3.37 \cdot 10^{-4}$	3.80	$7.23 \cdot 10^{-5}$	4.20	$1.33 \cdot 10^{-5}$	4.60	$2.11 \cdot 10^{-6}$
3.01	$1.31 \cdot 10^{-3}$	3.41	$3.25 \cdot 10^{-4}$	3.81	$6.95 \cdot 10^{-5}$	4.21	$1.28 \cdot 10^{-5}$	4.61	$2.01 \cdot 10^{-6}$
3.02	$1.26 \cdot 10^{-3}$	3.42	$3.13 \cdot 10^{-4}$	3.82	$6.67 \cdot 10^{-5}$	4.22	$1.22 \cdot 10^{-5}$	4.62	$1.92 \cdot 10^{-6}$
3.03	$1.22 \cdot 10^{-3}$	3.43	$3.02 \cdot 10^{-4}$	3.83	$6.41 \cdot 10^{-5}$	4.23	$1.17 \cdot 10^{-5}$	4.63	$1.83 \cdot 10^{-6}$
3.04	$1.18 \cdot 10^{-3}$	3.44	$2.91 \cdot 10^{-4}$	3.84	$6.15 \cdot 10^{-5}$	4.24	$1.12 \cdot 10^{-5}$	4.64	$1.74 \cdot 10^{-6}$
3.05	$1.14 \cdot 10^{-3}$	3.45	$2.80 \cdot 10^{-4}$	3.85	$5.91 \cdot 10^{-5}$	4.25	$1.07 \cdot 10^{-5}$	4.65	$1.66 \cdot 10^{-6}$
3.06	$1.11 \cdot 10^{-3}$	3.46	$2.70 \cdot 10^{-4}$	3.86	$5.67 \cdot 10^{-5}$	4.26	$1.02 \cdot 10^{-5}$	4.66	$1.58 \cdot 10^{-6}$
3.07	$1.07 \cdot 10^{-3}$	3.47	$2.60 \cdot 10^{-4}$	3.87	$5.44 \cdot 10^{-5}$	4.27	$9.77 \cdot 10^{-6}$	4.67	$1.51 \cdot 10^{-6}$
3.08	$1.04 \cdot 10^{-3}$	3.48	$2.51 \cdot 10^{-4}$	3.88	$5.22 \cdot 10^{-5}$	4.28	$9.34 \cdot 10^{-6}$	4.68	$1.43 \cdot 10^{-6}$
3.09	$1.00 \cdot 10^{-3}$	3.49	$2.42 \cdot 10^{-4}$	3.89	$5.01 \cdot 10^{-5}$	4.29	$8.93 \cdot 10^{-6}$	4.69	$1.37 \cdot 10^{-6}$
3.10	$9.68 \cdot 10^{-4}$	3.50	$2.33 \cdot 10^{-4}$	3.90	$4.81 \cdot 10^{-5}$	4.30	$8.54 \cdot 10^{-6}$	4.70	$1.30 \cdot 10^{-6}$
3.11	$9.35 \cdot 10^{-4}$	3.51	$2.24 \cdot 10^{-4}$	3.91	$4.61 \cdot 10^{-5}$	4.31	$8.16 \cdot 10^{-6}$	4.71	$1.24 \cdot 10^{-6}$
3.12	$9.04 \cdot 10^{-4}$	3.52	$2.16 \cdot 10^{-4}$	3.92	$4.43 \cdot 10^{-5}$	4.32	$7.80 \cdot 10^{-6}$	4.72	$1.18 \cdot 10^{-6}$
3.13	$8.74 \cdot 10^{-4}$	3.53	$2.08 \cdot 10^{-4}$	3.93	$4.25 \cdot 10^{-5}$	4.33	$7.46 \cdot 10^{-6}$	4.73	$1.12 \cdot 10^{-6}$
3.14	$8.45 \cdot 10^{-4}$	3.54	$2.00 \cdot 10^{-4}$	3.94	$4.07 \cdot 10^{-5}$	4.34	$7.12 \cdot 10^{-6}$	4.74	$1.07 \cdot 10^{-6}$
3.15	$8.16 \cdot 10^{-4}$	3.55	$1.93 \cdot 10^{-4}$	3.95	$3.91 \cdot 10^{-5}$	4.35	$6.81 \cdot 10^{-6}$	4.75	$1.02 \cdot 10^{-6}$
3.16	$7.89 \cdot 10^{-4}$	3.56	$1.85 \cdot 10^{-4}$	3.96	$3.75 \cdot 10^{-5}$	4.36	$6.50 \cdot 10^{-6}$	4.76	$9.68 \cdot 10^{-7}$
3.17	$7.62 \cdot 10^{-4}$	3.57	$1.78 \cdot 10^{-4}$	3.97	$3.59 \cdot 10^{-5}$	4.37	$6.21 \cdot 10^{-6}$	4.77	$9.21 \cdot 10^{-7}$
3.18	$7.36 \cdot 10^{-4}$	3.58	$1.72 \cdot 10^{-4}$	3.98	$3.45 \cdot 10^{-5}$	4.38	$5.93 \cdot 10^{-6}$	4.78	$8.76 \cdot 10^{-7}$
3.19	$7.11 \cdot 10^{-4}$	3.59	$1.65 \cdot 10^{-4}$	3.99	$3.30 \cdot 10^{-5}$	4.39	$5.67 \cdot 10^{-6}$	4.79	$8.34 \cdot 10^{-7}$
3.20	$6.87 \cdot 10^{-4}$	3.60	$1.59 \cdot 10^{-4}$	4.00	$3.17 \cdot 10^{-5}$	4.40	$5.41 \cdot 10^{-6}$	4.80	$7.93 \cdot 10^{-7}$
3.21	$6.64 \cdot 10^{-4}$	3.61	$1.53 \cdot 10^{-4}$	4.01	$3.04 \cdot 10^{-5}$	4.41	$5.17 \cdot 10^{-6}$	4.81	$7.55 \cdot 10^{-7}$
3.22	$6.41 \cdot 10^{-4}$	3.62	$1.47 \cdot 10^{-4}$	4.02	$2.91 \cdot 10^{-5}$	4.42	$4.94 \cdot 10^{-6}$	4.82	$7.18 \cdot 10^{-7}$
3.23	$6.19 \cdot 10^{-4}$	3.63	$1.42 \cdot 10^{-4}$	4.03	$2.79 \cdot 10^{-5}$	4.43	$4.71 \cdot 10^{-6}$	4.83	$6.83 \cdot 10^{-7}$
3.24	$5.98 \cdot 10^{-4}$	3.64	$1.36 \cdot 10^{-4}$	4.04	$2.67 \cdot 10^{-5}$	4.44	$4.50 \cdot 10^{-6}$	4.84	$6.49 \cdot 10^{-7}$
3.25	$5.77 \cdot 10^{-4}$	3.65	$1.31 \cdot 10^{-4}$	4.05	$2.56 \cdot 10^{-5}$	4.45	$4.29 \cdot 10^{-6}$	4.85	$6.17 \cdot 10^{-7}$
3.26	$5.57 \cdot 10^{-4}$	3.66	$1.26 \cdot 10^{-4}$	4.06	$2.45 \cdot 10^{-5}$	4.46	$4.10 \cdot 10^{-6}$	4.86	$5.87 \cdot 10^{-7}$
3.27	$5.38 \cdot 10^{-4}$	3.67	$1.21 \cdot 10^{-4}$	4.07	$2.35 \cdot 10^{-5}$	4.47	$3.91 \cdot 10^{-6}$	4.87	$5.58 \cdot 10^{-7}$
3.28	$5.19 \cdot 10^{-4}$	3.68	$1.17 \cdot 10^{-4}$	4.08	$2.25 \cdot 10^{-5}$	4.48	$3.73 \cdot 10^{-6}$	4.88	$5.30 \cdot 10^{-7}$
3.29	$5.01 \cdot 10^{-4}$	3.69	$1.12 \cdot 10^{-4}$	4.09	$2.16 \cdot 10^{-5}$	4.49	$3.56 \cdot 10^{-6}$	4.89	$5.04 \cdot 10^{-7}$
3.30	$4.83 \cdot 10^{-4}$	3.70	$1.08 \cdot 10^{-4}$	4.10	$2.07 \cdot 10^{-5}$	4.50	$3.40 \cdot 10^{-6}$	4.90	$4.79 \cdot 10^{-7}$
3.31	$4.66 \cdot 10^{-4}$	3.71	$1.04 \cdot 10^{-4}$	4.11	$1.98 \cdot 10^{-5}$	4.51	$3.24 \cdot 10^{-6}$	4.91	$4.55 \cdot 10^{-7}$
3.32	$4.50 \cdot 10^{-4}$	3.72	$9.96 \cdot 10^{-5}$	4.12	$1.89 \cdot 10^{-5}$	4.52	$3.09 \cdot 10^{-6}$	4.92	$4.33 \cdot 10^{-7}$
3.33	$4.34 \cdot 10^{-4}$	3.73	$9.57 \cdot 10^{-5}$	4.13	$1.81 \cdot 10^{-5}$	4.53	$2.95 \cdot 10^{-6}$	4.93	$4.11 \cdot 10^{-7}$
3.34	$4.19 \cdot 10^{-4}$	3.74	$9.20 \cdot 10^{-5}$	4.14	$1.74 \cdot 10^{-5}$	4.54	$2.81 \cdot 10^{-6}$	4.94	$3.91 \cdot 10^{-7}$
3.35	$4.04 \cdot 10^{-4}$	3.75	$8.84 \cdot 10^{-5}$	4.15	$1.66 \cdot 10^{-5}$	4.55	$2.68 \cdot 10^{-6}$	4.95	$3.71 \cdot 10^{-7}$
3.36	$3.90 \cdot 10^{-4}$	3.76	$8.50 \cdot 10^{-5}$	4.16	$1.59 \cdot 10^{-5}$	4.56	$2.56 \cdot 10^{-6}$	4.96	$3.52 \cdot 10^{-7}$
3.37	$3.76 \cdot 10^{-4}$	3.77	$8.16 \cdot 10^{-5}$	4.17	$1.52 \cdot 10^{-5}$	4.57	$2.44 \cdot 10^{-6}$	4.97	$3.35 \cdot 10^{-7}$
3.38	$3.62 \cdot 10^{-4}$	3.78	$7.84 \cdot 10^{-5}$	4.18	$1.46 \cdot 10^{-5}$	4.58	$2.32 \cdot 10^{-6}$	4.98	$3.18 \cdot 10^{-7}$
3.39	$3.49 \cdot 10^{-4}$	3.79	$7.53 \cdot 10^{-5}$	4.19	$1.39 \cdot 10^{-5}$	4.59	$2.22 \cdot 10^{-6}$	4.99	$3.02 \cdot 10^{-7}$

1. Two Variable Joint CDF, PMF and PDF
   (a) $F_{X,Y}(x,y) = P[X \le x, Y \le y] = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{X,Y}(u,v)\,dv\,du$
   (b) $P_{X,Y}(x,y) = P[X = x,\, Y = y]$
   (c) $f_{X,Y}(x,y) = \dfrac{\partial^2 F_{X,Y}(x,y)}{\partial x \partial y}$
2. Marginal PMFs and PDFs
   (a) Discrete: $P_X(x) = \sum_{y \in S_Y} P_{X,Y}(x,y)$ and $P_Y(y) = \sum_{x \in S_X} P_{X,Y}(x,y)$
   (b) Continuous: $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$ and $f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx$
3. Covariance and Correlation Coefficient
   (a) $E[X+Y] = E[X] + E[Y]$
   (b) $\text{Cov}[X,Y] = E\big[(X-\mu_X)(Y-\mu_Y)\big] = E[XY] - E[X]E[Y] = \mu_{XY} - \mu_X\mu_Y$
   (c) $\text{Var}[X+Y] = \text{Var}[X] + \text{Var}[Y] + 2\,\text{Cov}[X,Y]$
   (d) $\rho_{X,Y} = \dfrac{\text{Cov}[X,Y]}{\sqrt{\text{Var}[X]\text{Var}[Y]}} = \dfrac{\text{Cov}[X,Y]}{\sigma_X \sigma_Y}$
   (e) X and Y are said to be *uncorrelated* if $\text{Cov}[X,Y] = 0$.
4. Functions of Two Random Variables $W = g(X,Y)$
   (a) Discrete: $P_W(w) = \sum\sum_{g(x,\,y)=w} P_{X,Y}(x,y)$
   (b) Continuous: $F_W(w) = P[W \le w] = \iint_{g(x,\,y) \le w} f_{X,Y}(x,y)\,dx\,dy$
5. Expected Value of $W = g(X,Y)$
   (a) Discrete: $E\,[W] = \sum\sum_{(x,y) \in S_{X,Y}} g(x,y)\,P_{X,Y}(x,y)$
   (b) Continuous: $E\,[W] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y)\,f_{X,Y}(x,y)\,dx\,dy$
6. PDF of the Sum of Two Continuous Random Variables
   (a) The PDF of $W = X + Y$ is
   $$f_W(w) = \int_{-\infty}^{\infty} f_{X,Y}(x, w-x)\,dx = \int_{-\infty}^{\infty} f_{X,Y}(w-y, y)\,dy.$$
   (b) When X and Y are independent, the PDF of $W = X + Y$ is
   $$f_W(w) = \int_{-\infty}^{\infty} f_X(w-y)\,f_Y(y)\,dy = \int_{-\infty}^{\infty} f_X(x)\,f_Y(w-x)\,dx.$$

7. Conditioning a Random Variable Given an Event $B \subset S_X$ with $P[B] > 0$

   (a) Discrete: $$P_{X|B}(x) = \begin{cases} \dfrac{P_X(x)}{P[B]} & x \in B \\ 0 & \text{otherwise} \end{cases}$$

   (b) Continuous: $$f_{X|B}(x) = \begin{cases} \dfrac{f_X(x)}{P[B]} & x \in B \\ 0 & \text{otherwise} \end{cases}$$

8. Conditional Expected Value of a Function of a Random Variable Given an Event B

   (a) Discrete: $$E\big[g(X)|B\big] = \sum_{x \in B} g(x)\, P_{X|B}(x)$$

   (b) Continuous: $$E\big[g(X)|B\big] = \int_{-\infty}^{\infty} g(x)\, f_{X|B}(x)\, dx$$

9. Conditional Variance of a Random Variable Given an Event B

$$\text{Var}[X|B] = E[X^2|B] - \big(E[X|B]\big)^2$$

10. Conditional Joint PMF and PDF Given an Event $B \subset S_{X,Y}$ with $P[B] > 0$

   (a) $$P_{X,Y|B}(x,y) = \begin{cases} \dfrac{P_{X,Y}(x,y)}{P[B]} & (x,y) \in B \\ 0 & \text{otherwise} \end{cases}$$

   (b) $$f_{X,Y|B}(x,y) = \begin{cases} \dfrac{f_{X,Y}(x,y)}{P[B]} & (x,y) \in B \\ 0 & \text{otherwise} \end{cases}$$

11. Conditional Expected Value of $W = g(X,Y)$ Given an Event B

   (a) Discrete: $$E[W|B] = \sum_{x \in S_X} \sum_{y \in S_Y} g(x,y)\, P_{X,Y|B}(x,y)$$

   (b) Continuous: $$E[W|B] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y)\, f_{X,Y|B}(x,y)\, dx\, dy$$

12. Conditioning by a Random Variable

   (a) Conditional PMF: $$P_{X|Y}(x|y) = P[X = x|Y = y] = \frac{P_{X,Y}(x,y)}{P_Y(y)}$$

   Note: $P_{X,Y}(x,y) = P_{X|Y}(x|y)\, P_Y(y) = P_{Y|X}(y|x)\, P_X(x)$.

   (b) Conditional PDF: $$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

   Note: $f_{X,Y}(x,y) = f_{X|Y}(x|y)\, f_Y(y) = f_{Y|X}(y|x)\, f_X(x)$.

13. Conditional Expected Value of a Function $g(X,Y)$ Given $Y=y$

    (a) Discrete: $E[g(X,Y)|Y=y] = \sum_{x\in S_X} g(x,y)\,P_{X|Y}(x|y)$

    (b) Continuous: $E\left[g(X,Y)|Y=y\right] = \int_{-\infty}^{\infty} g(x,y)\,f_{X|Y}(x|y)\,dx$

14. N Independent Random Variables

    (a) Discrete: $P_{X_1,\ldots,X_n}(x_1,\ldots,x_n) = P_{X_1}(x_1)\,P_{X_2}(x_2)\,\cdots\,P_{X_N}(x_n)$

    (b) Continuous: $f_{X_1,\ldots,X_n}(x_1,\ldots,x_n) = f_{X_1}(x_1)\,f_{X_2}(x_2)\,\cdots\,f_{X_N}(x_n)$

15. Central Limit Theorem (Approximation)
    Let $W_n = X_1 + \cdots + X_n$ be the sum of n iid random variables, each with $E[X] = \mu_X$ and $\text{Var}[X] = \sigma_X^2$. The central limit theorem approximation to the CDF of W_n is

$$F_{W_N}(w) \approx \Phi\left(\frac{w - n\mu_X}{\sqrt{n\sigma_X^2}}\right).$$

16. De Moivre-Laplace Formula
    For a binomial (n,p) random variable K,

$$P[k_1 \le K \le k_2] \approx \Phi\left(\frac{k_2 + 0.5 - np}{\sqrt{np\,(1-p)}}\right) - \Phi\left(\frac{k_1 - 0.5 - np}{\sqrt{np\,(1-p)}}\right).$$

17. For the sample mean $M_n(X)$,

$$E\big[M_n(X)\big] = E[X] \quad \text{and} \quad \text{Var}\big[M_n(X)\big] = \frac{\text{Var}[X]}{n}.$$

18. Markov Inequality:
    For a random variable X such that $P[X<0]=0$ and a constant c,

$$P[X \ge c^2] \le \frac{E[X]}{c^2}.$$

19. Chebyshev Inequality: For an arbitrary random variable Y and constant $c>0$,

$$P\big[|Y - \mu_Y| \ge c\big] \le \frac{\text{Var}[Y]}{c^2}.$$

20. An estimate $\hat{R}$, of parameter r is unbiased if $E[\hat{R}] = r$; otherwise, $\hat{R}$ is biased.

21. If a sequence of unbiased estimates $\hat{R}_1, \hat{R}_2, \ldots$ of parameter r has mean square error $e_n = \text{Var}[\hat{R}_n]$ satisfying $\lim_{n\to\infty} e_n = 0$, then the sequence $\hat{R}_n$ is consistent.

22. The sample mean estimator $M_n(X)$ has mean square error

$$e_n = E\Big[\big(M_n(X) - E[X]\big)^2\Big] = \mathrm{Var}\big[M_n(X)\big] = \frac{\mathrm{Var}[X]}{n}.$$

23. Sample Variance:

(a) $V_n(X) = \dfrac{1}{n}\displaystyle\sum_{i=1}^{n}\big[X_i - M_n(X)\big]^2$ (biased estimator),

(b) $V_n'(X) = \dfrac{1}{n-1}\displaystyle\sum_{i=1}^{n}\big[X_i - M_n(X)\big]^2$ (unbiased estimator).

24. For any constant $c > 0$,

(a) $P\big[|M_n(X) - \mu_X| \geq c\big] \leq \dfrac{\mathrm{Var}[X]}{nc^2} = \alpha,$

(b) $P\big[|M_n(X) - \mu_X| < c\big] \geq 1 - \dfrac{\mathrm{Var}[X]}{nc^2} = 1 - \alpha.$

25. Given the indicator random variable X_A for an event A, $E[X_A] = P[A]$, $\mathrm{Var}[X_A] = P[A]\big(1 - P[A]\big)$, and

$$P\big[|\hat{P}_n(A) - P[A]| < c\big] \geq 1 - \frac{P[A]\big(1 - P[A]\big)}{nc^2}.$$

26. Let X be a Gaussian (μ, σ) random variable. A confidence interval estimate of μ of the form

$$M_n(X) - c \leq \mu \leq M_n(X) + c$$

has confidence coefficient $1 - \alpha$ where

$$\alpha/2 = Q\left(c\sqrt{n}/\sigma\right) = 1 - \Phi\left(c\sqrt{n}/\sigma\right).$$

27. If an experiment produces a random vector $\mathbf{X}$, the MAP hypothesis test is

$$\text{Discrete:}\quad \mathbf{x} \in A_0 \quad \text{if} \quad \frac{P_{\mathbf{X}|H_0}(\mathbf{x})}{P_{\mathbf{X}|H_1}(\mathbf{x})} \geq \frac{P[H_1]}{P[H_0]}\ ; \quad \mathbf{x} \in A_1 \text{ otherwise.}$$

$$\text{Continuous:}\quad \mathbf{x} \in A_0 \quad \text{if} \quad \frac{f_{\mathbf{X}|H_0}(\mathbf{x})}{f_{\mathbf{X}|H_1}(\mathbf{x})} \geq \frac{P[H_1]}{P[H_0]}\ ; \quad \mathbf{x} \in A_1 \text{ otherwise.}$$

28. If an experiment produces a random vector $\mathbf{X}$, the ML hypothesis test is

$$\text{Discrete:}\quad \mathbf{x} \in A_0 \quad \text{if} \quad \frac{P_{\mathbf{X}|H_0}(\mathbf{x})}{P_{\mathbf{X}|H_1}(\mathbf{x})} \geq 1\ ; \quad \mathbf{x} \in A_1 \text{ otherwise.}$$

$$\text{Continuous:}\quad \mathbf{x} \in A_0 \quad \text{if} \quad \frac{f_{\mathbf{X}|H_0}(\mathbf{x})}{f_{\mathbf{X}|H_1}(\mathbf{x})} \geq 1\ ; \quad \mathbf{x} \in A_1 \text{ otherwise.}$$

29. (a) The mean square error (MSE) is defined as $E\big[(X - \hat{X})^2\big]$.
    (b) The MMSE blind estimate of random variable X is $\hat{x}_B = E[X]$.
    (c) Given that $X \in A$, the MMSE estimate of X is $\hat{x}_A = E[X|A]$.
    (d) The MMSE estimate of random variable X given the observation $Y = y$ is

$$\hat{x}_M = E[X|Y = y].$$

30. The optimal linear mean square error estimator of X given Y is

$$\hat{X}_L(Y) = a^*Y + b^* \quad \text{where} \quad a^* = \frac{\mathrm{Cov}[X,Y]}{\mathrm{Var}[Y]} = \rho_{X,Y}\,\frac{\sigma_X}{\sigma_Y}, \quad b^* = \mu_X - a^*\mu_Y$$
$$\text{and} \quad e_L^* = E\Big[\big(X - \hat{X}_L(Y)\big)^2\Big] = \sigma_X^2\big[1 - (\rho_{X,Y})^2\big].$$

31. The MAP estimate of X given $Y = y$ is

$$\hat{x}_{\mathrm{MAP}}(y) = \arg\max_x \, f_{Y|X}(y|x)\, f_X(x) = \arg\max_x \, f_{X,Y}(x,y).$$

32. The ML estimate of X given $Y = y$ is

$$\hat{x}_{\mathrm{ML}}(y) = \arg\max_x \, f_{Y|X}(y|x).$$