Probability and Stochastic Processes:
A Friendly Introduction for Electrical and Computer Engineers
Edition 3
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Yates and Goodman 3e Solution Set: 6.2.2, 6.2.3, 6.2.5, 6.4.1, 6.4.2, 6.4.4, 6.4.5, 6.5.2, 6.5.3, 6.5.4, and 6.5.5

Problem 6.2.2 Solution

We start by finding the CDF $F_Y(y)$. Since $Y \geq 0$, $F_Y(y) = 0$ for $y < 0$. For $y \geq 0$,

$$\begin{aligned} F_Y(y) &= \mathrm{P}\left[|X| \leq y\right] \\ &= \mathrm{P}\left[-y \leq X \leq y\right] \\ &= \Phi(y) - \Phi(-y) = 2\Phi(y) - 1. \end{aligned} \tag{1}$$

It follows $f_Y(y) = 0$ for $y < 0$ and that for $y > 0$,

$$f_Y(y) = \frac{dF_Y(y)}{dy} = 2f_X(y) = \frac{2}{\sqrt{2\pi}} e^{-y^2/2}. \tag{2}$$

The complete expression for the CDF of Y is

$$f_Y(y) = \begin{cases} \frac{2}{\sqrt{2\pi}} e^{-y^2/2} & y \geq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

From the definition of the expected value,

$$\begin{aligned} \mathrm{E}\left[Y\right] &= \int_{-\infty}^{\infty} y f_Y(y)\, dy \\ &= \frac{2}{\sqrt{2\pi}} \int_0^{\infty} y e^{-y^2/2}\, dy = \sqrt{\frac{2}{\pi}}\, e^{-y^2/2} \Big|_0^{\infty} = \sqrt{\frac{2}{\pi}}. \end{aligned} \tag{4}$$

Problem 6.2.3 Solution

Note that T has the continuous uniform PDF

$$f_T(t) = \begin{cases} 1/15 & 60 \leq t \leq 75, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

The rider's maximum possible speed is $V = 3000/60 = 50$ km/hr while the rider's minimum speed is $V = 3000/75 = 40$ km/hr. For $40 \le v \le 50$,

$$F_V(v) = P\left[\frac{3000}{T} \le v\right] = P\left[T \ge \frac{3000}{v}\right]$$
$$= \int_{3000/v}^{75} \frac{1}{15}\,dt = \left.\frac{t}{15}\right|_{3000/v}^{75} = 5 - \frac{200}{v}. \tag{2}$$

Thus the CDF, and via a derivative, the PDF are

$$F_V(v) = \begin{cases} 0 & v < 40, \\ 5 - 200/v & 40 \le v \le 50, \\ 1 & v > 50, \end{cases} \qquad f_V(v) = \begin{cases} 0 & v < 40, \\ 200/v^2 & 40 \le v \le 50, \\ 0 & v > 50. \end{cases} \tag{3}$$

Problem 6.2.5 Solution

Since X is non-negative, $W = X^2$ is also non-negative. Hence for $w < 0$, $f_W(w) = 0$. For $w \ge 0$,

$$F_W(w) = P[W \le w] = P\left[X^2 \le w\right]$$
$$= P[X \le w]$$
$$= 1 - e^{-\lambda\sqrt{w}}. \tag{1}$$

Taking the derivative with respect to w yields $f_W(w) = \lambda e^{-\lambda\sqrt{w}}/(2\sqrt{w})$. The complete expression for the PDF is

$$f_W(w) = \begin{cases} \frac{\lambda e^{-\lambda\sqrt{w}}}{2\sqrt{w}} & w \ge 0, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Problem 6.4.1 Solution

Since $0 \le X \le 1$, and $0 \le Y \le 1$, we have $0 \le V \le 1$. This implies $F_V(v) = 0$ for