Probability and Stochastic Processes:
A Friendly Introduction for Electrical and Computer Engineers
Edition 3
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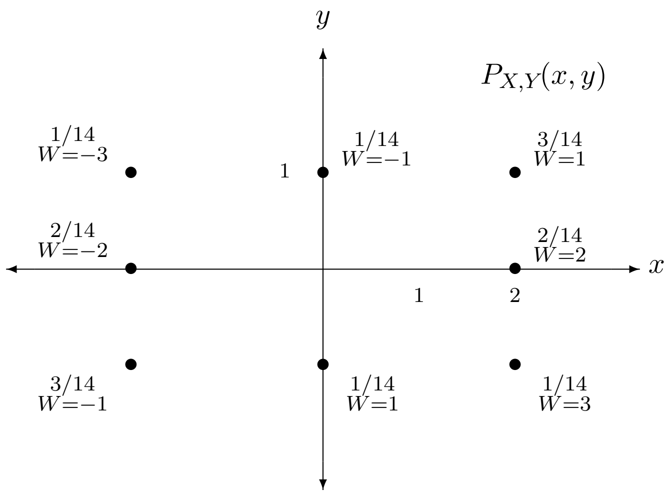
Yates and Goodman 3e Solution Set: 6.1.1, 6.1.3, 6.1.5, and 6.1.6

Problem 6.1.1 Solution

In this problem, it is helpful to label possible points X, Y along with the corresponding values of $W = X - Y$. From the statement of Problem 6.1.1,



To find the PMF of W, we simply add the probabilities associated with each possible value of W:

$$P_W(-3) = P_{X,Y}(-2,1) = 1/14, \tag{1}$$
$$P_W(-2) = P_{X,Y}(-2,0) = 2/14, \tag{2}$$
$$P_W(-1) = P_{X,Y}(-2,-1) + P_{X,Y}(0,1) = 4/14, \tag{3}$$
$$P_W(1) = P_{X,Y}(0,-1) + P_{X,Y}(2,1) = 4/14, \tag{4}$$
$$P_W(2) = P_{X,Y}(2,0) = 2/14, \tag{5}$$
$$P_W(3) = P_{X,Y}(2,1) = 1/14. \tag{6}$$

For all other values of w, $P_W(w) = 0$. A table for the PMF of W is

w	-3	-2	-1	1	2	3
$P_W(w)$	$1/14$	$2/14$	$4/14$	$4/14$	$2/14$	$1/14$

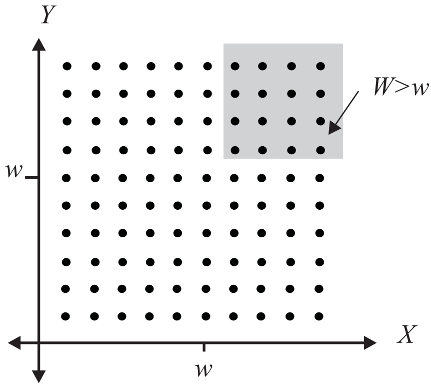
Problem 6.1.3 Solution

This is basically a trick problem. It looks like this problem should be in Section 6.5 since we have to find the PMF of the sum $L = N + M$. However, this problem is an special case since N and M are both binomial with the same success probability $p = 0.4$.

In this case, N is the number of successes in 100 independent trials with success probability $p = 0.4$. M is the number of successes in 50 independent trials with success probability $p = 0.4$. Thus $L = M + N$ is the number of successes in 150 independent trials with success probability $p = 0.4$. We conclude that L has the binomial $(n = 150, p = 0.4)$ PMF

$$P_L(l) = \binom{150}{l}(0.4)^l(0.6)^{150-l}. \tag{1}$$

Problem 6.1.5 Solution



The x, y pairs with nonzero probability are shown in the figure. For $w = 0, 1, \ldots, 10$, we observe that

$$\begin{aligned} \mathrm{P}[W > w] &= \mathrm{P}[\min(X, Y) > w] \\ &= \mathrm{P}[X > w, Y > w] \\ &= 0.01(10 - w)^2. \end{aligned} \tag{1}$$

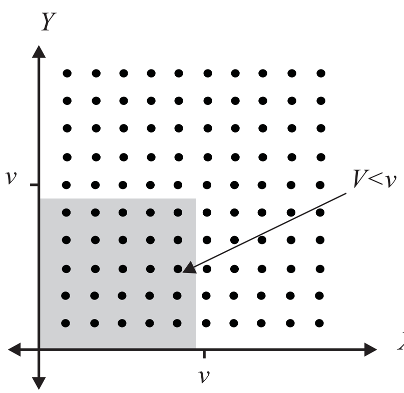
To find the PMF of W, we observe that for $w = 1, \ldots, 10$,

$$\begin{aligned} P_W(w) &= \mathrm{P}[W > w - 1] - \mathrm{P}[W > w] \\ &= 0.01[(10 - w - 1)^2 - (10 - w)^2] = 0.01(21 - 2w). \end{aligned} \tag{2}$$

The complete expression for the PMF of W is

$$P_W(w) = \begin{cases} 0.01(21 - 2w) & w = 1, 2, \ldots, 10, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Problem 6.1.6 Solution



The x, y pairs with nonzero probability are shown in the figure. For $v = 1, \ldots, 11$, we observe that

$$\begin{aligned} \mathrm{P}[V < v] &= \mathrm{P}[\max(X, Y) < v] \\ &= \mathrm{P}[X < v, Y < v] \\ &= 0.01(v - 1)^2. \end{aligned} \tag{1}$$

To find the PMF of V, we observe that for $v = 1, \ldots, 10$,

$$\begin{aligned} P_V(v) &= \mathrm{P}\left[V < v+1\right] - \mathrm{P}\left[V < v\right] \\ &= 0.01[v^2 - (v-1)^2] \\ &= 0.01(2v-1). \end{aligned} \tag{2}$$

The complete expression for the PMF of V is

$$P_V(v) = \begin{cases} 0.01(2v-1) & v = 1, 2, \ldots, 10, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$