Probability and Stochastic Processes:
A Friendly Introduction for Electrical and Computer Engineers
Edition 3
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Yates and Goodman 3e Solution Set: 3.6.2, 3.6.4, 3.6.7, 3.7.5, 3.7.6, 3.7.8, and 3.8.8

Problem 3.6.2 Solution

From the solution to Problem 3.4.2, the PMF of X is

$$P_X(x) = \begin{cases} 0.2 & x = -1, \\ 0.5 & x = 0, \\ 0.3 & x = 1, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

(a) The PMF of $V = |X|$ satisfies

$$P_V(v) = \mathrm{P}\left[|X| = v\right] = P_X(v) + P_X(-v). \tag{2}$$

In particular,

$$P_V(0) = P_X(0) = 0.5, \tag{3}$$
$$P_V(1) = P_X(-1) + P_X(1) = 0.5. \tag{4}$$

The complete expression for the PMF of V is

$$P_V(v) = \begin{cases} 0.5 & v = 0, \\ 0.5 & v = 1, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

(b) From the PMF, we can construct the staircase CDF of V.

$$F_V(v) = \begin{cases} 0 & v < 0, \\ 0.5 & 0 \le v < 1, \\ 1 & v \ge 1. \end{cases} \tag{6}$$

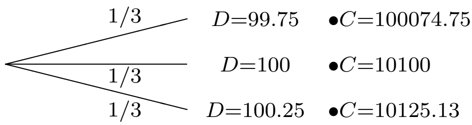
(c) From the PMF $P_V(v)$, the expected value of V is

$$E[V] = \sum_v P_V(v) = 0(1/2) + 1(1/2) = 1/2. \tag{7}$$

You can also compute E[V] directly by using Theorem 3.10.

Problem 3.6.4 Solution

A tree for the experiment is



Thus C has three equally likely outcomes. The PMF of C is

$$P_C(c) = \begin{cases} 1/3 & c = 100{,}074.75, 10{,}100, 10{,}125.13, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Problem 3.6.7 Solution

(a) A student is properly counted with probability p, independent of any other student being counted. Hence, we have 70 Bernoulli trials and N is a binomial $(70, p)$ random variable with PMF

$$P_N(n) = \binom{70}{n} p^n (1-p)^{70-n}. \tag{1}$$

(b) There are two ways to find this. The first way is to observe that

$$\begin{aligned} P[U = u] = P[N = 70 - u] &= P_N(70 - u) \\ &= \binom{70}{70-u} p^{70-u} (1-p)^{70-(70-u)} \\ &= \binom{70}{u} (1-p)^u p^{70-u}. \end{aligned} \tag{2}$$

We see that U is a binomial $(70, 1-p)$. The second way is to argue this directly since U is counting overlooked students. If we call an overlooked student a "success" with probability $1-p$, then U, the number of successes in n trials, is binomial $(70, 1-p)$.

(c)

$$\begin{aligned} \mathrm{P}[U \geq 2] &= 1 - \mathrm{P}[U < 2] \\ &= 1 - (P_U(0) + P_U(1)) \\ &= 1 - (p^{70} + 70(1-p)p^{69}). \end{aligned} \tag{3}$$

(d) The binomial $(n = 70, 1-p)$ random variable U has $\mathrm{E}[U] = 70(1-p)$. Solving $70(1-p) = 2$ yields $p = 34/35$.

Problem 3.7.5 Solution

Given the distributions of D, the waiting time in days and the resulting cost, C, we can answer the following questions.

(a) The expected waiting time is simply the expected value of D.

$$\mathrm{E}[D] = \sum_{d=1}^{4} d \cdot P_D(d) = 1(0.2) + 2(0.4) + 3(0.3) + 4(0.1) = 2.3. \tag{1}$$

(b) The expected deviation from the waiting time is

$$\mathrm{E}[D - \mu_D] = \mathrm{E}[D] - \mathrm{E}[\mu_d] = \mu_D - \mu_D = 0. \tag{2}$$

(c) C can be expressed as a function of D in the following manner.

$$C(D) = \begin{cases} 90 & D = 1, \\ 70 & D = 2, \\ 40 & D = 3, \\ 40 & D = 4. \end{cases} \tag{3}$$

(d) The expected service charge is

$$\mathrm{E}[C] = 90(0.2) + 70(0.4) + 40(0.3) + 40(0.1) = 62 \text{ dollars}. \tag{4}$$

Problem 3.7.6 Solution

FALSE: For a counterexample, suppose

$$P_X(x) = \begin{cases} 0.5 & x = 0.1, 0.5 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

In this case, $\mathrm{E}[X] = 0.5(0.1 + 0.5) = 0.3$ so that $1/\mathrm{E}[X] = 10/3$. On the other hand,

$$\mathrm{E}[1/X] = 0.5\left(\frac{1}{0.1} + \frac{1}{0.5}\right) = 0.5(10 + 2) = 6. \tag{2}$$

Problem 3.7.8 Solution

Since our phone use is a geometric random variable M with mean value $1/p$,

$$P_M(m) = \begin{cases} (1-p)^{m-1}p & m = 1, 2, \ldots, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

For this cellular billing plan, we are given no free minutes, but are charged half the flat fee. That is, we are going to pay 15 dollars regardless and \$1 for each minute we use the phone. Hence $C = 15 + M$ and for $c \geq 16$, $\mathrm{P}[C = c] = \mathrm{P}[M = c - 15]$. Thus we can construct the PMF of the cost C

$$P_C(c) = \begin{cases} (1-p)^{c-16}p & c = 16, 17, \ldots \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Since $C = 15 + M$, the expected cost per month of the plan is

$$\mathrm{E}[C] = \mathrm{E}[15 + M] = 15 + \mathrm{E}[M] = 15 + 1/p. \tag{3}$$

In Problem 3.7.7, we found that that the expected cost of the plan was

$$\mathrm{E}[C] = 20 + [(1-p)^{30}]/(2p). \tag{4}$$

In comparing the expected costs of the two plans, we see that the new plan is better (i.e. cheaper) if

$$15 + 1/p \leq 20 + [(1-p)^{30}]/(2p). \tag{5}$$

A simple plot will show that the new plan is better if $p \leq p_0 \approx 0.2$.

Problem 3.8.8 Solution

Given that

$$Y = \frac{1}{\sigma_x}(X - \mu_X), \tag{1}$$

we can use the linearity property of the expectation operator to find the mean value

$$\mathrm{E}[Y] = \frac{\mathrm{E}[X - \mu_X]}{\sigma_X} = \frac{\mathrm{E}[X] - \mathrm{E}[X]}{\sigma_X} = 0. \tag{2}$$

Using the fact that $\mathrm{Var}[aX + b] = a^2 \mathrm{Var}[X]$, the variance of Y is found to be

$$\mathrm{Var}[Y] = \frac{1}{\sigma_X^2} \mathrm{Var}[X] = 1. \tag{3}$$