Probability and Stochastic Processes:
A Friendly Introduction for Electrical and Computer Engineers
Edition 3
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Yates and Goodman 3e Solution Set: 3.3.3, 3.3.5, 3.3.6, 3.3.10, 3.3.14, and 3.3.16

Problem 3.3.3 Solution

(a) Each paging attempt is an independent Bernoulli trial with success probability p. The number of times K that the pager receives a message is the number of successes in n Bernoulli trials and has the binomial PMF

$$P_K(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & k = 0, 1, \ldots, n, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

(b) Let R denote the event that the paging message was received at least once. The event R has probability

$$\mathrm{P}[R] = \mathrm{P}[B > 0] = 1 - \mathrm{P}[B = 0] = 1 - (1-p)^n. \tag{2}$$

To ensure that $\mathrm{P}[R] \geq 0.95$ requires that $n \geq \ln(0.05)/\ln(1-p)$. For $p = 0.8$, we must have $n \geq 1.86$. Thus, $n = 2$ pages would be necessary.

Problem 3.3.5 Solution

Whether a hook catches a fish is an independent trial with success probability h. The the number of fish hooked, K, has the binomial PMF

$$P_K(k) = \begin{cases} \binom{m}{k} h^k (1-h)^{m-k} & k = 0, 1, \ldots, m, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Problem 3.3.6 Solution

(a) Let X be the number of times the frisbee is thrown until the dog catches it and runs away. Each throw of the frisbee can be viewed as a Bernoulli trial in which a success occurs if the dog catches the frisbee an runs away. Thus, the experiment ends on the first success and X has the geometric PMF

$$P_X(x) = \begin{cases} (1-p)^{x-1}p & x = 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

(b) The child will throw the frisbee more than four times iff there are failures on the first 4 trials which has probability $(1-p)^4$. If $p = 0.2$, the probability of more than four throws is $(0.8)^4 = 0.4096$.

Problem 3.3.10 Solution

Since an average of $T/5$ buses arrive in an interval of T minutes, buses arrive at the bus stop at a rate of $1/5$ buses per minute.

(a) From the definition of the Poisson PMF, the PMF of B, the number of buses in T minutes, is

$$P_B(b) = \begin{cases} (T/5)^b e^{-T/5}/b! & b = 0, 1, \ldots, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

(b) Choosing $T = 2$ minutes, the probability that three buses arrive in a two minute interval is

$$P_B(3) = (2/5)^3 e^{-2/5}/3! \approx 0.0072. \tag{2}$$

(c) By choosing $T = 10$ minutes, the probability of zero buses arriving in a ten minute interval is

$$P_B(0) = e^{-10/5}/0! = e^{-2} \approx 0.135. \tag{3}$$

(d) The probability that at least one bus arrives in T minutes is

$$\mathrm{P}[B \geq 1] = 1 - \mathrm{P}[B = 0] = 1 - e^{-T/5} \geq 0.99. \tag{4}$$

Rearranging yields $T \geq 5 \ln 100 \approx 23.0$ minutes.

Problem 3.3.14 Solution

(a) We can view whether each caller knows the birthdate as a Bernoulli trial. As a result, L is the number of trials needed for 6 successes. That is, L has a Pascal PMF with parameters $p = 0.75$ and $k = 6$ as defined by Definition 3.7. In particular,

$$P_L(l) = \begin{cases} \binom{l-1}{5}(0.75)^6(0.25)^{l-6} & l = 6, 7, \ldots, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

(b) The probability of finding the winner on the tenth call is

$$P_L(10) = \binom{9}{5}(0.75)^6(0.25)^4 \approx 0.0876. \tag{2}$$

(c) The probability that the station will need nine or more calls to find a winner is

$$\begin{aligned} \mathrm{P}[L \geq 9] &= 1 - \mathrm{P}[L < 9] \\ &= 1 - P_L(6) - P_L(7) - P_L(8) \\ &= 1 - (0.75)^6[1 + 6(0.25) + 21(0.25)^2] \approx 0.321. \end{aligned} \tag{3}$$

Problem 3.3.16 Solution

(a) Note that $L_1 = l$ if there are l consecutively arriving planes landing in l minutes, which has probability p^l, followed by an idle minute without an arrival, which has probability $1 - p$. Thus,

$$P_{L_1}(l) = \begin{cases} p^l(1-p) & l = 0, 1, 2, \ldots \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

(b) As indicated, $W = 10$ if there are ten consecutive takeoffs in ten minutes. This occurs if there are no landings for 10 minutes, which occurs with probability $(1-p)^{10}$. Thus $\mathrm{P}[W = 10] = (1-p)^{10}$.

(c) The key is to recognize that each takeoff is a "success" with success probability $1-p$. Moreover, in each one minute slot, a success occurs with probability $1-p$, independent of the result of the trial in any other one minute slot. Since your plane is the tenth in line, your plane takes off when the tenth success occurs. Thus W is a Pascal $(10, 1-p)$ random variable and has PMF

$$\begin{aligned} P_W(w) &= \binom{w-1}{9}(1-p)^{10}(1-(1-p))^{w-10} \\ &= \binom{w-1}{9}(1-p)^{10}p^{w-10}. \end{aligned} \tag{2}$$