Probability and Stochastic Processes:
A Friendly Introduction for Electrical and Computer Engineers
Edition 3
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Yates and Goodman 3e Solution Set: 3.2.2, 3.2.4, 3.2.5, 3.2.6, 3.4.2, 3.4.3, and 3.4.7

Problem 3.2.2 Solution

(a) We must choose c to make the PMF of V sum to one.

$$\sum_{v=1}^{4} P_V(v) = c(1^2 + 2^2 + 3^2 + 4^2) = 30c = 1. \tag{1}$$

Hence $c = 1/30$.

(b) Let $U = \{u^2 | u = 1, 2, \ldots\}$ so that

$$P[V \in U] = P_V(1) + P_V(4) = \frac{1}{30} + \frac{4^2}{30} = \frac{17}{30}. \tag{2}$$

(c) The probability that V is even is

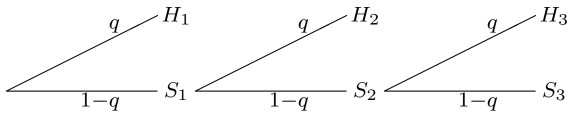
$$P[V \text{ is even}] = P_V(2) + P_V(4) = \frac{2^2}{30} + \frac{4^2}{30} = \frac{2}{3}. \tag{3}$$

(d) The probability that $V > 2$ is

$$P[V > 2] = P_V(3) + P_V(4) = \frac{3^2}{30} + \frac{4^2}{30} = \frac{5}{6}. \tag{4}$$

Problem 3.2.4 Solution

(a) The starting point is to draw a tree diagram. On swing i, let H_i denote the event that Casey hits a home run and S_i the event that he gets a strike. The tree is



We see that Casey hits a home run as long as he does not strike out. That is,

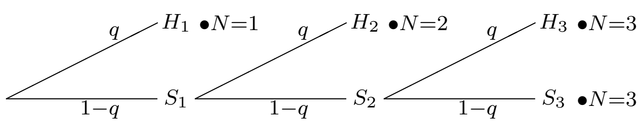
$$\mathrm{P}[H] = 1 - \mathrm{P}[S_3] = 1 - (1-q)^3 = 1 - (0.95)^3. \tag{1}$$

A second way to solve this problem is to observe that Casey hits a home run if he hits a homer on any of his three swings, $H = H_1 \cup H_2 \cup H_3$. Since $\mathrm{P}[H_i] = (1-q)^{i-1}q$ and since the events H_i are mutually exclusive,

$$\mathrm{P}[H] = \mathrm{P}[H_1] + \mathrm{P}[H_2] + \mathrm{P}[H_3] = 0.05(1 + 0.95 + 0.95^2) \tag{2}$$

This can be simplified to the first answer.

(b) Now we label the outcomes of the tree with the sample values of N:



From the tree,

$$P_N(n) = \begin{cases} q & n = 1, \\ (1-q)q & n = 2, \\ (1-q)^2 & n = 3, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Problem 3.2.5 Solution

(a) To find c, we apply the constraint $\sum_n P_N(n) = 1$, yielding

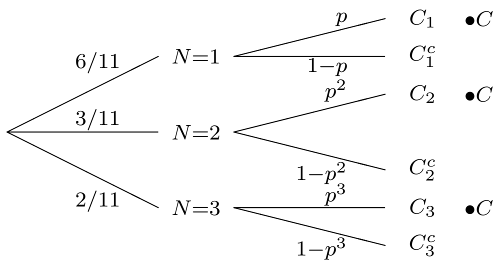
$$1 = \sum_{n=1}^{3} \frac{c}{n} = c\left(1 + \frac{1}{2} + \frac{1}{3}\right) = c\left(\frac{11}{6}\right). \tag{1}$$

Thus $c = 6/11$.

(b) The probability that N is odd is

$$P[N \text{ is odd}] = P_N(1) + P_N(3) = c\left(1 + \frac{1}{3}\right) = c\left(\frac{4}{3}\right) = \frac{24}{33}. \tag{2}$$

(c) We can view this as a sequential experiment: first we divide the file into N packets and then we check that all N packets are received correctly. In the second stage, we could specify how many packets are received correctly; however, it is sufficient to just specify whether the N packets are all received correctly or not. Using C_n to denote the event that n packets are transmitted and received correctly, we have

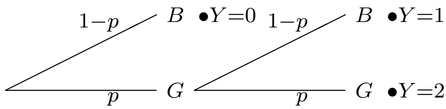


We see that

$$\begin{aligned} P[C] &= P[C_1] + P[C_2] + P[C_3] \\ &= \frac{6p}{11} + \frac{3p^2}{11} + \frac{2p^3}{11} = \frac{p(6 + 3p + 2p^2)}{11}. \end{aligned} \tag{3}$$

Problem 3.2.6 Solution

Using B (for Bad) to denote a miss and G (for Good) to denote a successful free throw, the sample tree for the number of points scored in the 1 and 1 is

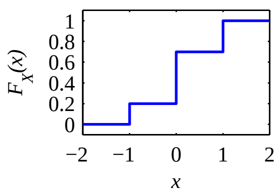


From the tree, the PMF of Y is

$$P_Y(y) = \begin{cases} 1-p & y=0, \\ p(1-p) & y=1, \\ p^2 & y=2, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Problem 3.4.2 Solution

(a) The given CDF is shown in the diagram below.



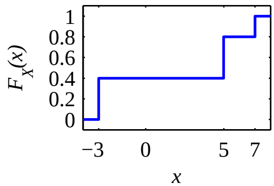
$$F_X(x) = \begin{cases} 0 & x < -1, \\ 0.2 & -1 \le x < 0, \\ 0.7 & 0 \le x < 1, \\ 1 & x \ge 1. \end{cases} \tag{1}$$

(b) The corresponding PMF of X is

$$P_X(x) = \begin{cases} 0.2 & x=-1, \\ 0.5 & x=0, \\ 0.3 & x=1, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Problem 3.4.3 Solution

(a) Similar to the previous problem, the graph of the CDF is shown below.



$$F_X(x) = \begin{cases} 0 & x < -3, \\ 0.4 & -3 \le x < 5, \\ 0.8 & 5 \le x < 7, \\ 1 & x \ge 7. \end{cases} \tag{1}$$

(b) The corresponding PMF of X is

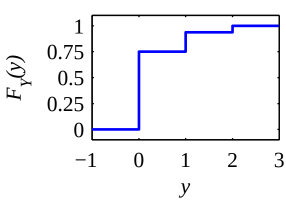
$$P_X(x) = \begin{cases} 0.4 & x = -3 \\ 0.4 & x = 5 \\ 0.2 & x = 7 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Problem 3.4.7 Solution

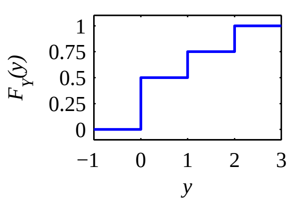
In Problem 3.2.6, we found the PMF of Y. This PMF, and its corresponding CDF are

$$P_Y(y) = \begin{cases} 1-p & y = 0, \\ p(1-p) & y = 1, \\ p^2 & y = 2, \\ 0 & \text{otherwise,} \end{cases} \qquad F_Y(y) = \begin{cases} 0 & y < 0, \\ 1-p & 0 \le y < 1, \\ 1-p^2 & 1 \le y < 2, \\ 1 & y \ge 2. \end{cases} \tag{1}$$

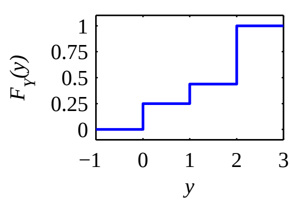
For the three values of p, the CDF resembles



$p = 1/4$



$p = 1/2$



$p = 3/4$