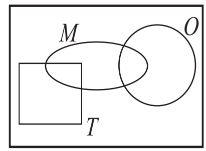
Probability and Stochastic Processes:
A Friendly Introduction for Electrical and Computer Engineers
Edition 3

Roy D. Yates and David J. Goodman

Yates and Goodman 3e Solution Set: 1.1.2, 1.2.2, 1.3.1, 1.3.4, 1.3.6, 1.4.1, 1.4.3, 1.4.5, 1.4.6, 1.5.1, and 1.5.2

Problem 1.1.2 Solution

Based on the Venn diagram on the right, the answers are mostly fairly straightforward. The only trickiness is that a pizza is either Tuscan (T) or Neapolitan (N) so $\{N, T\}$ is a partition but they are not depicted as a partition. Specifically, the event N is the region of the Venn diagram outside of the "square block" of event T. If this is clear, the questions are easy.



(a) Since $N = T^c$, $N \cap M \neq \phi$. Thus N and M are not mutually exclusive.

(b) Every pizza is either Neapolitan (N), or Tuscan (T). Hence $N \cup T = S$ so that N and T are collectively exhaustive. Thus its also (trivially) true that $N \cup T \cup M = S$. That is, R, T and M are also collectively exhaustive.

(c) From the Venn diagram, T and O are mutually exclusive. In words, this means that Tuscan pizzas never have onions or pizzas with onions are never Tuscan. As an aside, "Tuscan" is a fake pizza designation; one shouldn't conclude that people from Tuscany actually dislike onions.

(d) From the Venn diagram, $M \cap T$ and O are mutually exclusive. Thus Gerlanda's doesn't make Tuscan pizza with mushrooms and onions.

(e) Yes. In terms of the Venn diagram, these pizzas are in the set $(T \cup M \cup O)^c$.

Problem 1.2.2 Solution

(a) The sample space of the experiment is

$$S = \{aaa, aaf, afa, faa, ffa, faf, aff, fff\}. \tag{1}$$

(b) The event that the circuit from Z fails is

$$Z_F = \{aaf, aff, faf, fff\}. \tag{2}$$

The event that the circuit from X is acceptable is

$$X_A = \{aaa, aaf, afa, aff\}. \tag{3}$$

(c) Since $Z_F \cap X_A = \{aaf, aff\} \neq \phi$, Z_F and X_A are not mutually exclusive.

(d) Since $Z_F \cup X_A = \{aaa, aaf, afa, aff, faf, fff\} \neq S$, Z_F and X_A are not collectively exhaustive.

(e) The event that more than one circuit is acceptable is

$$C = \{aaa, aaf, afa, faa\}. \tag{4}$$

The event that at least two circuits fail is

$$D = \{ffa, faf, aff, fff\}. \tag{5}$$

(f) Inspection shows that $C \cap D = \phi$ so C and D are mutually exclusive.

(g) Since $C \cup D = S$, C and D are collectively exhaustive.

Problem 1.3.1 Solution

(a) A and B mutually exclusive and collectively exhaustive imply $\mathrm{P}[A]+\mathrm{P}[B] = 1$. Since $\mathrm{P}[A] = 3\,\mathrm{P}[B]$, we have $\mathrm{P}[B] = 1/4$.

(b) Since $\mathrm{P}[A \cup B] = \mathrm{P}[A]$, we see that $B \subseteq A$. This implies $\mathrm{P}[A \cap B] = \mathrm{P}[B]$. Since $\mathrm{P}[A \cap B] = 0$, then $\mathrm{P}[B] = 0$.

(c) Since it's always true that $\mathrm{P}[A \cup B] = \mathrm{P}[A] + \mathrm{P}[B] - \mathrm{P}[AB]$, we have that

$$\mathrm{P}[A] + \mathrm{P}[B] - \mathrm{P}[AB] = \mathrm{P}[A] - \mathrm{P}[B]. \tag{1}$$

This implies $2\,\mathrm{P}[B] = \mathrm{P}[AB]$. However, since $AB \subset B$, we can conclude that $2\,\mathrm{P}[B] = \mathrm{P}[AB] \leq \mathrm{P}[B]$. This implies $\mathrm{P}[B] = 0$.

Problem 1.3.4 Solution

(a) FALSE. Since $P[A] = 1 - P[A^c] = 2\,P[A^c]$ implies $P[A^c] = 1/3$.

(b) FALSE. Suppose $A = B$ and $P[A] = 1/2$. In that case,

$$P[AB] = P[A] = 1/2 > 1/4 = P[A]\,P[B]. \tag{1}$$

(c) TRUE. Since $AB \subseteq A$, $P[AB] \leq P[A]$, This implies

$$P[AB] \leq P[A] < P[B]. \tag{2}$$

(d) FALSE: For a counterexample, let $A = \phi$ and $P[B] > 0$ so that $A = A \cap B = \phi$ and $P[A] = P[A \cap B] = 0$ but $0 = P[A] < P[B]$.

Problem 1.3.6 Solution

A sample outcome indicates whether the cell phone is handheld (H) or mobile (M) and whether the speed is fast (F) or slow (W). The sample space is

$$S = \{HF, HW, MF, MW\}. \tag{1}$$

The problem statement tells us that $P[HF] = 0.2$, $P[MW] = 0.1$ and $P[F] = 0.5$. We can use these facts to find the probabilities of the other outcomes. In particular,

$$P[F] = P[HF] + P[MF]. \tag{2}$$

This implies

$$P[MF] = P[F] - P[HF] = 0.5 - 0.2 = 0.3. \tag{3}$$

Also, since the probabilities must sum to 1,

$$\begin{aligned} P[HW] &= 1 - P[HF] - P[MF] - P[MW] \\ &= 1 - 0.2 - 0.3 - 0.1 = 0.4. \end{aligned} \tag{4}$$

Now that we have found the probabilities of the outcomes, finding any other probability is easy.

(a) The probability a cell phone is slow is

$$P[W] = P[HW] + P[MW] = 0.4 + 0.1 = 0.5. \tag{5}$$

(b) The probability that a cell hpone is mobile and fast is $P[MF] = 0.3$.

(c) The probability that a cell phone is handheld is

$$P[H] = P[HF] + P[HW] = 0.2 + 0.4 = 0.6. \tag{6}$$

Problem 1.4.1 Solution

Each question requests a conditional probability.

(a) Note that the probability a call is brief is

$$P[B] = P[H_0 B] + P[H_1 B] + P[H_2 B] = 0.6. \tag{1}$$

The probability a brief call will have no handoffs is

$$P[H_0|B] = \frac{P[H_0 B]}{P[B]} = \frac{0.4}{0.6} = \frac{2}{3}. \tag{2}$$

(b) The probability of one handoff is $P[H_1] = P[H_1 B] + P[H_1 L] = 0.2$. The probability that a call with one handoff will be long is

$$P[L|H_1] = \frac{P[H_1 L]}{P[H_1]} = \frac{0.1}{0.2} = \frac{1}{2}. \tag{3}$$

(c) The probability a call is long is $P[L] = 1 - P[B] = 0.4$. The probability that a long call will have one or more handoffs is

$$\begin{aligned} P[H_1 \cup H_2|L] &= \frac{P[H_1 L \cup H_2 L]}{P[L]} \\ &= \frac{P[H_1 L] + P[H_2 L]}{P[L]} = \frac{0.1 + 0.2}{0.4} = \frac{3}{4}. \end{aligned} \tag{4}$$

Problem 1.4.3 Solution

Since the 2 of clubs is an even numbered card, $C_2 \subset E$ so that $P[C_2E] = P[C_2] = 1/3$. Since $P[E] = 2/3$,

$$P[C_2|E] = \frac{P[C_2E]}{P[E]} = \frac{1/3}{2/3} = 1/2. \tag{1}$$

The probability that an even numbered card is picked given that the 2 is picked is

$$P[E|C_2] = \frac{P[C_2E]}{P[C_2]} = \frac{1/3}{1/3} = 1. \tag{2}$$

Problem 1.4.5 Solution

The first generation consists of two plants each with genotype yg or gy. They are crossed to produce the following second generation genotypes, $S = \{yy, yg, gy, gg\}$. Each genotype is just as likely as any other so the probability of each genotype is consequently 1/4. A pea plant has yellow seeds if it possesses at least one dominant y gene. The set of pea plants with yellow seeds is

$$Y = \{yy, yg, gy\}. \tag{1}$$

So the probability of a pea plant with yellow seeds is

$$P[Y] = P[yy] + P[yg] + P[gy] = 3/4. \tag{2}$$

Problem 1.4.6 Solution

Define D as the event that a pea plant has two dominant y genes. To find the conditional probability of D given the event Y, corresponding to a plant having yellow seeds, we look to evaluate

$$P[D|Y] = \frac{P[DY]}{P[Y]}. \tag{1}$$

Note that $P[DY]$ is just the probability of the genotype yy. From Problem 1.4.5, we found that with respect to the color of the peas, the genotypes yy, yg, gy, and gg were all equally likely. This implies

$$P[DY] = P[yy] = 1/4 \qquad P[Y] = P[yy, gy, yg] = 3/4. \tag{2}$$

Thus, the conditional probability can be expressed as

$$P[D|Y] = \frac{P[DY]}{P[Y]} = \frac{1/4}{3/4} = 1/3. \quad (3)$$

Problem 1.5.1 Solution

From the table we look to add all the mutually exclusive events to find each probability.

(a) The probability that a caller makes no hand-offs is

$$P[H_0] = P[LH_0] + P[BH_0] = 0.1 + 0.4 = 0.5. \quad (1)$$

(b) The probability that a call is brief is

$$P[B] = P[BH_0] + P[BH_1] + P[BH_2] = 0.4 + 0.1 + 0.1 = 0.6. \quad (2)$$

(c) The probability that a call is long or makes at least two hand-offs is

$$\begin{aligned} P[L \cup H_2] &= P[LH_0] + P[LH_1] + P[LH_2] + P[BH_2] \\ &= 0.1 + 0.1 + 0.2 + 0.1 = 0.5. \end{aligned} \quad (3)$$

Problem 1.5.2 Solution

(a) From the given probability distribution of billed minutes, M, the probability that a call is billed for more than 3 minutes is

$$\begin{aligned} P[L] &= 1 - P[\text{3 or fewer billed minutes}] \\ &= 1 - P[B_1] - P[B_2] - P[B_3] \\ &= 1 - \alpha - \alpha(1-\alpha) - \alpha(1-\alpha)^2 \\ &= (1-\alpha)^3 = 0.57. \end{aligned} \quad (1)$$

(b) The probability that a call will billed for 9 minutes or less is

$$P[\text{9 minutes or less}] = \sum_{i=1}^{9} \alpha(1-\alpha)^{i-1} = 1 - (0.57)^3. \quad (2)$$