Initia Rerum Mathematicarum Metaphysica

G.W. Leibniz, 1715
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Since the distinguished mathematician Christian Wolff, in his Latin Course in Mathematics, recently touched on and illustrated, in his own way, certain meditations of mine about the Analysis of Axioms and about the nature of similarity (see Act. Erudit A. 1714), it seemed good to set out some things pertaining to this that I worked out mentally some time ago, lest they be lost; from which it can be understood that there is a certain Analytical art broader than Mathematics, from which Mathematical science borrows certain of its most beautiful Methods. I would like to start, therefore, a little deeper:

If one supposes that multiple states of things exist, involving nothing opposed, they are said to exist ***simultaneously***. And so we say that what happened in the past year and the present year are not simultaneous, for they involve opposed states of the same thing.

If one of those that are not simultaneous involves the reason for another, the former is considered ***prior*** *and the latter* ***posterior***. My prior state involves the reason that the posterior exists. And since my prior state, because of the connection of all things, also involves the prior state of other things, hence my prior state also involves the reason for the posterior state of other things and so also is prior to the state of other things. And so, *whatever exists is either simultaneous with, or prior to, or posterior to, another existing thing*.

Time *is the order of existing of those that are not simultaneous*. And so it is also the general order of changes when the species of the changes is not in view.

Duration *is the magnitude of time.* If the magnitude of time is diminished continuously and uniformly [aequabiliter], time vanishes in a Moment, which has no magnitude.

Space *is the order of coexisting*, or the order of existing among things that are simultaneous.

According to either order (of time or of space) things are deemed ***closer together or farther apart*** *according as more or fewer things are co-required to understand the order between them.* Hence two points are closer whose interpolating [points] maximally determined by them give something simpler.[1] Such a maximally determined interpolating thing is the simplest path from the one to the other, minimal and at the same time maximally uniform [aequabilis], namely a line, of which a lesser one intervenes between closer points.

Extension *is the magnitude of space.* Frequently extension is wrongly conflated with the extensum itself, and considered in the likeness of a substance.

If the magnitude of space is diminished uniformly and continuously, it vanishes in a point which has no magnitude.

Situs *is the mode of coexistence.* Thus it involves not only quantity, but also quality.

Quantity *or Magnitude is that which can be known in things only by co-presence (or simultaneous perception).*

Thus it cannot be known what a foot or a cubit is, unless we actually have something like a measure that can then be applied to other things. Nor indeed can a foot be sufficiently explained by any definition which does not in fact involve again some such thing. For even if we said that a foot is twelve inches, there is the same question about an inch, and we do not gain any further insight from it, nor can one say whether the notion of an inch or a foot is prior in nature, since it is arbitrary which one we want to take as the basis.

But ***Quality*** *is that which can be known in things when they are observed individually* and co-presence is not needed. Such are attributes that are explained by a definition or through the various modifications that they involve.

Equals are of the same quantity.

Similars are of the same quality. Hence if two similars are distinct, they cannot be distinguished except by co-presence.

Hence it is clear for example that two equiangular Triangles have proportional sides and vice versa. For if the sides are proportional, the triangles are certainly similar, since they are determined in a similar manner. Further, in every triangle the sum of the angles is the same, since it is equal to two right angles; therefore it is necessary that the ratio of the corresponding angles to the sum in the one is what it is in the other; otherwise by that very fact the one triangle could be distinguished from the other, indeed viewed in itself

[1] Cf. Leibniz's definition of the line determined by two points as the set of points which are unique of their situs to the two given points.

or individually. Thus is easily demonstrated what otherwise requires many convolutions.

Homogeneous things are those to which equals can be given that are similar to each other. Let there be A and B, and if L can be taken equal to A and M equal to B such that L and M are similar, then A and B will be called Homogeneous.

Hence I also usually say that Homogeneous things are those that can be rendered similar by transformation, like a curve to a line. Namely if A is transformed into its equal L, it can be made similar to B or to M into which B is assumed to be transformed.

That is said to *be-in* some locus or to be an *ingredient* of some thing, which, when the [locus or] thing is posited, is understood to be posited immediately by that very fact, so that, indeed, there is no need for inferences. Thus when we posit some finite line, we posit its extrema, its parts.

Something homogeneous that is-in, is called a *Part*, and that which it is-in is called the *Whole*, or i.e. a part is a homogeneous ingredient.

A *common boundary* is that which is-in two things not having a common part. Whenever these are understood to be parts of the same whole, that common boundary is called a *Section* of the whole.

Hence it is clear that a Boundary is not homogeneous with what is bounded, nor is a Section homogeneous with what is sectioned.

Time and Moment, Space and Point, Boundary and Bounded, even though they are not Homogeneous, are nevertheless ***homogonous**, in that one can pass into the other by a continuous change*.

A *locus* which is said to *be-in* another locus, we understand to be Homogonous, because if it were a part or equal to a part of it, it would be not only homogonous but also homogeneous. Even though an angle is at a point, it is not in a point, otherwise magnitude would be thought to be in a point.

If a part of one is equal to another whole, the latter is called *Lesser*, the former *Greater*.

And so *the Whole is greater than a part*. Let A be the whole, B a part; I say that A is greater than B, because a part of A (namely B) is equal to the whole B. The matter can also be set out in a Syllogism, whose Major proposition is a definition and Minor proposition is identical [identica]:[A]

> Whatever is equal to a part of A, that is less than A, by definition;
> B is equal to a part of A, namely itself, by hypothesis;
> Therefore B is less than A.

Hence we see that demonstrations ultimately are resolved into two indemonstrables: Definitions or ideas, and primitive, namely identical, propositions, such as this is: B is B, each thing is equal to itself, and infinite others of this kind.

Motion is change of situs.

That *moves* in which there is a change of situs and simultaneously the reason for the change.

A movable thing is homogonous with an extensum, for even a point is understood to be movable.

A *path* is the continuous successive locus of a movable thing.

A *track* is the locus of a movable thing which it occupies at some moment. Hence the track of a boundary is a section of the path which the boundary describes, when of course the movable thing does not proceed in its own tracks.

A *movable* is said to *proceed in its own tracks* when any point of it except the boundary continuously follows in the place of another point of the same movable thing.

But if a Movable is not assumed to move in this way, then[2]

A *Curve* is the path of a point.

A *Surface* is the path of a Curve.

An *Expanse* [Amplum] or *Space*, or in common terms a solid, is the path of a surface.

The magnitudes of paths by which a point describes a curve, a curve a surface, and a surface an expanse, are called *length*, *width*, and *depth*. They are called dimensions, and in Geometry it is shown that there are only three.

That *has Width* of which there is an extended section, or which is bounded by something extended.

That *has Depth* which does not bound an extensum, or which cannot be the section of an extensum; indeed, in depth there is something more than could be a boundary.

A curve is the last bounding extensum.

An expanse is the last Bounded extensum.

Similarity or dissimilarity in an expanse or space is recognized based on the boundaries, and so an *expanse*, since it is something more than could be a boundary, is everywhere similar inside. Expanses whose entire extremities coincide, are congruent, are similar, will be coincident, congruent, similar. It is the same in the plane, which is a surface uniform or similar to itself inside, and in a *line*, which is a curve similar to itself inside.

[2] A marginal insertion ends here abruptly, with space below. A connecting line indicates it is somewhat likely that Leibniz intended the text to continue directly with the next sentence.

The *entire extremity* in Extensa having width can be called the *Ambit*. Thus the ambit of a circle is the circumference, the ambit of a sphere is the spherical surface.

A *point* (that is, of space) is the simplest locus, or a locus of no other locus.

Absolute space is the fullest locus, or the locus of all loci.

From one point nothing prosults.[3]

From two points something new prosults, namely any point unique of its situs to them, and the locus of all these, that is, the line which passes through the two proposed points.

From three points a *plane* prosults, that is, the locus of all points unique of their situs to three points not falling on the same line.

From four points not falling on the same plane prosults *absolute Space*. For every point is unique of its situs to four points not falling in the same plane.

I use the word *prosulting* to indicate a new idea, when from certain posited things something else is determined, by that very fact, which is unique of its relation to them. By relation here we understand situs.

Time can be continued to infinity. For since a whole of time is similar to a part, it relates to another time as the part relates to it, and so it is understood to continue in another greater time.

Similarly also *solid space or expanse can be continued to infinity*, inasmuch as a part of it can be taken similar to the whole. And hence a plane and also a line continue to infinity. In the same way one shows that space, just like a line and also time, and the continuum in general, can be subdivided to infinity. For in a line and in time a part is similar to the whole and so can be sectioned in the same ratio as the whole, and even if there are extensa in which a part is not similar to the whole, still they can be transformed into such, and sectioned in the same ratio as those into which they were transformed.

It also follows from these things that for *any* movement one can assume a faster and slower one in a given ratio: indeed when a rigid radius is driven around a center, the movements of the points are as their distances from the center, and so the speeds may be varied as the lines.

The *estimation* of magnitudes is twofold, imperfect and perfect; imperfect, when we say that something is greater than or less than something else, although they are neither homogeneous nor have a proportion to each other, as when someone says that a Curve is greater than a point, or a surface than

[3]The Latin *prosulto* may be Leibniz's coinage, or a technical usage he invents for a very rare word.

a curve. In this sense Euclid said that an Angle of contact[4] is less than any rectilinear angle, even though in reality there is no comparison between these completely distinct kinds,[B] since they are not homogeneous, nor is it possible to pass from the one to the other by a continuous change. In perfect estimations between homogeneous things the following rule obtains, that in passing continuously from one extreme to another, one passes through all intermediates; but this does not obtain in imperfect estimations because what is called a mediate is heterogeneous, and so, passing from a given acute angle to a right angle, one does not pass through the Angles of a semicircle, or radii to the circumference, even though it is said to be less than a right angle and greater than any acute angle, for here the Greater is taken improperly for that which falls inside another.

There are various relations according to quantity. Thus two lines can have the Relation to each other that their sum is equal to a constant line. And infinitely many pairs of lines can be given having this relation to each other, namely x and y such that $x + y = a$, if for instance a is as 10, then x and y can be as 1 and 9, as 2 and 8, as 3 and 7, as 4 and 6, as 5 and 5, as 6 and 4, as 7 and 3, as 8 and 2, as 9 and 1. But infinitely many fractions can also be taken under 10 that satisfy it. Thus too, such a relation is given between two lines x and y that their squares taken together equal the given square of the line a, so we have $xx + yy = aa$; and infinitely many pairs of such lines can also be given, and this is the relation of the cosine [sinus complementi] to the sine in the Circle and conversely, indeed setting the one as x, the other is y and the radius is a. And infinitely many such relations can be created, as many as the species of curves that can be described in the plane. For instance, if x are the abscissae of the directrix line, y will be the mutually parallel ordinates attached to the abscissae which end at the Curve.

But the simplest of all Relations is that which is called *Ratio* or *Proportion*, and it is the Relation of two homogeneous quantities that arises from them alone without assuming a third homogeneous one. For instance when y is to x as a number is to unity, or $y = nx$, in which case, setting x as the abscissae and y as the ordinates, the locus is a line; I mean the locus or Curve at[5] which the ordinates terminate. It is also clear from this that, if the equation were a local one, of whatever degree, such as $lx^3 + my^3 + nxxy + pxyy$ where l, m, n, p are pure numbers, then the locus for which this is the equation would be a line, and the ratio of x and y would be given.

[4] Cf. [insert reference about angles of contact].

[5] The Latin here is not really legible, but is plausibly ‘ad quam’.

Let two lines be given which are compared with each other in some manner. For example let the lesser be subtracted from the greater as many times as possible, and the remainder again from the lesser, and then again the remainder [subtracted] from that which is subtracted as many times as possible, until either exhaustion is attained, the last subtrahend being a common Measure, if the quantities are commensurable, or else one has a law of progression to infinity if they are incommensurable. And the series of Quotient numbers will be the same when the proportion is the same. Namely, if a is to b as

$$l + \cfrac{1}{m + \cfrac{1}{n + \cfrac{1}{p + etc.}}}$$

is to unity, then l, m, n, p, etc. will be the series of quotient numbers. For example, if a is 17 and b is 5, the series will only consist of three, l, m, n, which numbers are 3, 2, 2.[6] If a and b are parts of a line sectioned with an extreme and middle ratio, the greater a will be to the lesser b as $1+\frac{1}{1+\frac{1}{1+\frac{1}{1+etc.}}}$ to unity; the quotients will be units, and their series will go on to infinity. Thus any lines a and b will be to each other as $\frac{l}{1}+\frac{m}{2}+\frac{n}{4}+\frac{p}{8}+\frac{q}{16}+$ etc. to 1, setting l,m,n,p,q to be 0 or 1, and this series either terminates or is periodic when the numbers are commensurable.

It follows from this that similar curves are in the ratio of Homologous lines, similar surfaces are as the squares of homologous lines, similar solids are as their cubes. Let there be two similar Extensa A and L, and homologous, homogeneous things B and M. Since A together with B, or $A;B$,[7] is similar to L together with M, or $L;M$, the ratio of A to B will be the same as that of L to M, otherwise $A;L$ could be distinguished from $B;M$ in some way other than by copresence; indeed different numbers expressing the ratios would result. Therefore, by permuting, A will be to L as B to M, as was proposed. Thus it is shown that circles are as the squares of the diameters, spheres as the cubes of the diameters. The circles (spheres) will be A and L, the homologous squares (homologous cubes) will be B and M.

From this it is clear that *Number* in general, whole, fractional, rational, surd, ordinal, transcendental, can be defined by the general notion that it is what is homogeneous to Unity, or that which relates to Unity as a line to a

[6]The MS has 2, 3, 2, presumably by mistake.
[7]The MS here is not legible.

line. It is also clear that, if the Ratio of a to b were considered as the number which is to Unity as the line a to the line b, then the Ratio itself would be homogeneous to Unity, and Unity, moreover, represents the Ratio of equality.

It should also be noted that the whole doctrine of Algebra is an application to quantities of the Combinatorial Art, or of the abstract doctrine of Forms in the mind, which is Characteristic in general and pertains to Metaphysics. Thus the product of the multiplication of $a+b+c+etc.$ by $l+m+n+etc.$ is nothing but the sum of all pairs of letters from different series [ordo], and the product of three series multiplied together, $a+b+c+etc.$ by $l+m+n+etc.$ by $s+t+v+etc.$, will be the sum of all triples of letters from different series; and from other operations other forms result.

Hence in calculus it is useful to observe not only the law of homogeneous things, but also that of justice,[C] that whatever things behave in the same way in the givens or assumptions, also behave in the same way in what is sought or what emerges, and which allows one to treat them conveniently in the same way while performing operations;[8] and in general one may conclude that when the givens come in order, so also the things sought come in order. Hence also follows the *Law of Continuity* which I was the first to set forth, by which it happens that the law of things at rest is like [quasi] a species of the law of things being in motion, the law of equals like a species of the law of unequals, as the law of Curvilinear things is like a species of the law of rectilinear things.[D] This always takes place whenever a genus ends in an opposite quasi-species. And here pertains that argument Geometers used to marvel at by which, from that which is assumed to be something, one directly proves that it is not or conversely, or by which, something taken as if a species, is found to be opposite or disparate. And that is the privilege of the continuous; *Continuity* can in fact be found in time, extension, qualities, motions, and every *transition of nature*, which never happens by a jump.

Situs is a certain relation of coexisting among multiple things, and it is known through other coexisting things, intermediates, that is, things which have a simpler relation to the former ones of coexisting.[E]

We know to coexist not only things which are perceived simultaneously, but also things which we perceive successively, if it be assumed only that, during the transition from perception of the one to perception of the other, either the earlier has not perished, or the later has not been born. From the first hypothesis it follows that both coexist now when we reach the later; from the second it follows that both existed when we left the earlier.

[8] Leibniz inserted the last clause after writing the subsequent sentence, and its syntax is not entirely compatible with that of the first part.

Now there is a certain order in the transition of perception, when one passes from one to another through other things. And this can be called a *path*. But as this order can vary in infinitely many ways, it is necessary that one is the simplest, which of course is by proceeding through the determined intermediates according to the very nature of the thing, that is, through those which relate to either extreme most simply. For if it did not exist, there would be no order, no basis for distinguishing [ratio discernendi] in the coexistence of things, since one could go from given to given through anything.[F] And this is the minimal path from one to another, whose magnitude is called *distance*.

In order to understand these things better, we shall now abstract our mind from what can be seen in the individual things whose distance is being treated; thus we will consider them as if a plurality is not to be seen in the individuals, i.e. we will consider them as Points. For something is a *point* in which nothing else coexisting with it is posited, so that whatever is in it, is it.

Then the path of a point will be a *curve*, which of course does not have *width* because a section of it, which happens at a point, does not have length.

From one given point nothing else is determined. But with two points given, the simplest path from the one to the other is determined, which we call a *Line*.

(1) Hence it follows *first* that a line is the minimal [path] from point to point, i.e. its magnitude is the distance of the points.

(2) *Second,* that a line is uniform [aequabilem] between its extremes. For nothing is assumed from which a reason for a variation could be provided.

(3) And so it must be the case that one locus of a point moving along it cannot be distinguished from another, setting aside reference to the extremes. Hence also a part of a line is a line, and so inside it is self-similar everywhere, and neither can two parts be distinguished from each other when their extremes cannot be distinguished.

(4) It also follows that, supposing the extremes are similar or congruent or coincident, the lines themselves are similar, congruent, or coincident. But the extremes are always similar. And so any two lines are similar, and also a part to the whole.

(5) *Third,* it follows from the definition that a line proceeds through points unique of their relation to the two given points, which rule [ratio] is maximally determined. There must be such [points], otherwise no new determined thing would result from two givens. And if there were some other point relating in the same way to A and B taken together, as proposed, there would be no reason why that simplest, determined path would proceed through one rather

than the other. This is also clear from the preceding, when[9] we showed that the line is determined given the extremes, or the lines coincide if the extremes coincide.

(6) *Fourth,* it follows that a line behaves in the same way toward all regions,[10] neither can it be concave and convex like a curve, since from two assumed points A and B, no reason could be given for a distinction.

(7) And hence if any two points L and M are assumed outside the line, which relate in the same way to two points of the line collectively, or so that L relates to A and B as M relates to A and B, then they will also relate in the same way to the whole line, or L will relate to the line through A and B as M relates to it.

(7)[11] It is also clear that a *rigid* line, or one whose points do not change situs among themselves, cannot be moved while two points on it remain unmoved, for otherwise multiple points would be given relating in the same way to the two unmoved points, namely the point at which the movable point was, as well as that to which it is transferred.

(8) It follows in turn that all other points which do not fall on the line passing through A and B, i.e. which are not in alignment with A and B, are movable with the situs to A and B preserved, A and B remaining unmoved, since the line is the locus of all points relating uniquely to A and B; therefore any others can vary, and indeed to all sides, since the line relates in the same way to all sides.

(9) And so if a rigid Extensum moves with two points remaining unmoved, all its points at rest will fall on the line passing through the unmoved points, and any movable point will describe a circle around the same line as an axis.

Given three points not falling on the same Line, what is determined from this is a *plane*. Let A, B, C be points not falling on the same line; from the points A, B the line through A and B is determined, from the points C and B the line through C and B is determined. From any point of the line through A and B, together with any point of the line through C and B, a new line is determined, and hence given A, B, C, infinitely many lines are determined, the locus of which is called a *plane*.[G]

(1) And so *first,* a plane is the minimum between its extremes. Indeed its ambit does not consist of a line, because a line does not enclose space, else a part of the line would be dissimilar to the whole. Therefore, the ambit being

[9]The Latin here, 'quia dum,' has a superfluous conjunction.

[10]Leibniz deleted 'partes' and inserted 'plagas'.

[11]The MS has two paragraphs numbered with (7). This second one has the number deleted and rewritten with a marginal note about the duplication.

given, three points not falling on the same line are given, therefore from the given ambit alone the intervening [interceptum] plane is determined. Therefore it is the minimum.

(2) *Second,* a plane is uniform between its extremes, because no reason for variation can be deduced from this its origin.

(3) Whence it follows that a plane is self-similar inside, thus a point which moves in it does not distinguish the place where it is from another place except with respect to the extremes. Neither can a part of the plane be discerned from another part except through the extremes.

(4) It follows furthermore that planes whose ambits are similar or congruent or coincident are themselves similar or congruent or coincident.

(5) *Third,* it is clear from the definition of a plane that it is the locus of all points unique [of their relation] to three given points.

(6) *Fourth,* it follows that a plane relates in the same way on both sides, and thus has neither concavity nor convexity.

(7) And so to a point relating in whatever way to A, B, C together, or to the plane determined by them, another corresponding point can be given relating in the same way to these three points taken together, since there is no reason for a distinction.

(8) Now a plane is endowed with width, since it can be sectioned by a straight line passing through a given pair of its points. And so a section of it has length; and that whose section has length, itself has width.

Given four points not falling on the same plane, there results depth, or that in which something can be taken that is not a boundary, i.e. which cannot be common to it and some other that isn't a part immersed in it.

Let there be four points A, B, C, D. [See Figure 1.] From this are given six lines AB, AC, AD, BC, BD, CD; but AB, AC, AD suffice, since the remaining three arise from these. With these three lines prosulting, all of their points also prosult, as well as the lines conjoining any two points of distinct lines, and so also the locus of all these lines. Furthermore we have also four planes, through $A.B.C$, through $A.B.D$, through $A.C.D$, through $B.C.D$, of which any two have a common line as a section; for example the planes through $A.B.C$ and $B.C.D$ have as a common section the line through $B.C$.These four planes enclose a space with four planar triangles ABC, ABD, ACD, BCD, which form the ambit of the space. And each line EF connecting two points E and F of the two planes ABC, BCD has all points G inside this space, so that from a point G, interposed between the extremes of the line, no line can be drawn out

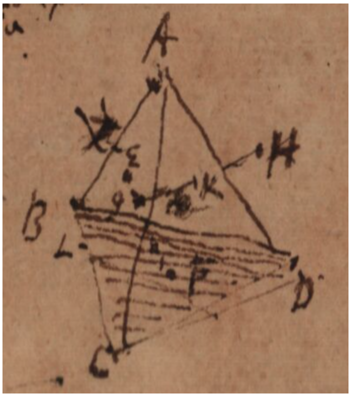

Figure 1

that does not fall upon the ambit. Now[12] those *enclose space* which constitute a full ambit, namely one such that any curve drawn in a part of the ambit, when it reaches an extreme of that part, can be continued in another part of the ambit. For example, the line AL drawn on the part ABC of the ambit, when it reaches its extreme L, could not be continued in another part of the ambit if the triangle BCD were missing, which together with the other three triangles ABC, ABD, ACD completes the task of enclosing space.[H] Take any other point H; I say that the line GH, extended if necessary, meets one of the four triangles, such as ABD, at K; the plane through E, F, H will intersect the planes ABD, ACD, BCD; these three lines[I] constitute a triangle, of which two sides will fall on two of the three aforementioned triangles, and the point G will fall inside this triangle. Therefore any line passing through G, and therefore the line GH, will intersect one of these two sides; therefore the line GH meets one of the triangles ABD, ACD, BCD at K. In the same way it will be shown to meet one of the other three triangles at the other extreme as well. But we will show the matter more briefly. Suppose that line is extended as far as necessary to a point H which is farther from the point G than all points of the triangles ABC, ABD, ACD, BCD. From the point H let normals be directed to the four planes, and likewise from the point G; from these planes some plane will be found with respect to which the normals of both are not on the same side; some one of these planes must intersect GH[13] within a triangle

[12] These sentences are inserted in the right margin of the manuscript.

[13] In the manuscript, it appears that Leibniz originally had GH and then wrote GK over top of

of which the plane is a continuation.[14]

it. The correction seems to have been a mistake.

[14]This paragraph is an alternative argument to what Leibniz gave in the previous paragraph. The manuscript here shows significant editorial work. Leibniz has crossed out portions of the previous paragraph, circled the previous paragraph, and added this alternative argument together with the connecting clause 'But we will show the matter more briefly'. In the previous paragraph, Leibniz inserted what appears to be an alternative version of the first sentence, namely 'I claim that the line GH, extended if necessary, falls on one of these planes', which we conjecture he added upon realizing his new argument only shows that GH would intersect one of the *planes*, and not necessarily fall in one of the triangles.

Notes

[A]An 'identical' proposition, for Leibniz, is one which expresses an identity, i.e. a tautology relating to a thing's identity.

[B]Here Leibniz acknowledges that there can be a sense in which an entity (the angle of contact) is less than every finite magnitude, while indicating clearly that this sense is not the plain sense of 'reality'. This passage is therefore relevant to the interpretation of Leibniz's views on infinitesimals.

[C]Leibniz's *lex justitiae*, or law of justice, is also called the law of correspondence, or the law of equipollence.

[D]Equals are a special case of things comparable in quantity. Curves are a special case of polygons, if you allow treating the limit, or terminus, of a set or process, like the others.

[E]Here Leibniz seems to allude to an incremental procedure of decomposition of a figure, by which some of its parts, such as lines and circles, are constructed individually, and the whole is constructed as a combination of these.

[F]Leibniz is aware of the impossibility of assigning a unique straight line between the poles of a sphere. He might assess that case using the concept here by concluding that indeed, the north and south pole being given, no reason could be given for choosing any particular third point.

[G]Leibniz considers a locus of *lines*, rather than points, only rarely. Notice also that Leibniz says the third lines are determined (given choices of the points). Whether they are determined absolutely, relative to the original three points, or only contextually here, relative to the choices of additional points, is not entirely clear; although Leibniz does believe that the points on the line through A and B are determined if the line itself is determined (for example). One wants to suggest that the *plane* is determined, although the additional points and the lines they determine are not themselves fully determined.

[H]An acutely modern topological claim.

[I]That is, the lines formed by the intersection of the plane EFH with each of the planes ABD, ACD, BCD.