Name: Solutions

Worksheet Supplement - Complex Numbers

1) Evaluate and write in the form, a + bi (LT: 7d)

a) $(1+3i)(5-i) = 5+15i-i-3i^2$

$= \boxed{8+14i}$

b) $\dfrac{2+5i}{-3+7i}\left(\dfrac{-3-7i}{-3-7i}\right) = \dfrac{-6-14i-15i-35i^2}{9-49i^2}$

$= \dfrac{-29+29i}{58}$

$= \boxed{-\frac{1}{2}+\frac{1}{2}i}$

c) $(2i)^3 = 2^3 i^3$

$= 8i^2(i)$

$= \boxed{-8i}$

d) $\sqrt{-4}\sqrt{-16}$

$=(2i)(4i)$

$= \boxed{-8}$

2) Find all solutions of the equation (LT: 7d):

a) $16x^2+9=0$

$16x^2=-9$

$x^2=-\dfrac{9}{16}$

$x = \pm\sqrt{-\dfrac{9}{16}}$

$= \boxed{\pm\frac{3}{4}i}$

b) $2x^2+x+1=0$

$x = \dfrac{-1\pm\sqrt{1^2-4(2)(1)}}{2(2)} = \dfrac{-1\pm\sqrt{-7}}{4} = \boxed{-\frac{1}{4}\pm\frac{\sqrt{7}}{4}i}$

c) $x^2+\dfrac{1}{3}x+\dfrac{1}{9}=0$

$x = \dfrac{-\frac{1}{3}\pm\sqrt{\frac{1}{9}-4\left(\frac{1}{9}\right)}}{2} = \dfrac{-\frac{1}{3}\pm\sqrt{-\frac{3}{9}}}{2} = \dfrac{-\frac{1}{3}\pm\sqrt{\frac{1}{3}}\,i}{2} = \boxed{-\frac{1}{6}\pm\frac{\sqrt{3}}{6}i}$

3) Write the number in polar form with argument between 0 and 2π. (LT: 7a, 7b)

a) $-5+5i$

$\theta = \dfrac{3\pi}{4}$

$r = \sqrt{25+25}$

$\boxed{5\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)}$

b) $2-2\sqrt{3}i$

$\tan\theta = -\dfrac{2\sqrt{3}}{2}$

$\theta = \dfrac{5\pi}{3}$

$r = \sqrt{4+12} = 4$

$\boxed{4\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)}$

c) $6i$

$\theta = \dfrac{\pi}{2}$

$r = 6$

$\boxed{6\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)}$

4) Find polar forms for zw, z/w, and 1/z, with argument between 0 and 2π. (LT: 7b, 7d)

a) $z = 1 + \sqrt{3}i,\ w = \sqrt{3} + i$

$Z = 2 \text{cis} \frac{\pi}{3}$ $w = 2 \text{cis} \frac{\pi}{6}$

$Zw = 4 \text{cis} \frac{\pi}{2}$ $\frac{1}{Z} = \frac{1}{2} \text{cis} -\frac{\pi}{3}$ or $\frac{1}{2} \text{cis} \frac{5\pi}{3}$

$\frac{Z}{W} = 1 \text{cis} \frac{\pi}{6}$

$$\boxed{\begin{aligned} Zw &= 4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) \\ \frac{Z}{W} &= \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} \\ \frac{1}{Z} &= \frac{1}{2}\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) \end{aligned}}$$

b) $z = 2\sqrt{3} - 2i,\ w = 6i$

$Z = 4 \text{cis} \frac{11\pi}{6}$ $\frac{1}{Z} = \frac{1}{4} \text{cis} -\frac{11\pi}{6}$ or $\frac{1}{4} \text{cis} \frac{\pi}{6}$

$W = 6 \text{cis} \frac{\pi}{2}$

$Zw = 24 \text{cis} \frac{14\pi}{6}$

$\frac{Z}{W} = \frac{2}{3} \text{cis} \frac{8\pi}{6}$

$$\boxed{\begin{aligned} Zw &= 24\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) \\ \frac{Z}{W} &= \frac{2}{3}\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right) \\ \frac{1}{Z} &= \frac{1}{4}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \end{aligned}}$$

5) Find the indicated power using De Moivre's Theorem. Write in a + bi form. (LT: 7d)

a) $(1+i)^{16}$

$1+i = \sqrt{2} \text{cis} \frac{\pi}{4}$

$(1+i)^{16} = (\sqrt{2})^{16} \text{cis} 4\pi$

$= 2^8 \text{cis} 4\pi = \boxed{256}$

b) $\left(\sqrt{3} - i\right)^5$

$\sqrt{3} - i = 2 \text{cis} \frac{11\pi}{6}$

$(\sqrt{3} - i)^5 = 2^5 \text{cis} \frac{55\pi}{6}$

$= 32\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$

$= 32\left(-\frac{\sqrt{3}}{2} - i\left(\frac{1}{2}\right)\right)$

$= \boxed{-16\sqrt{3} - 16i}$

6) Find the indicated roots in polar form. When possible without a calculator, write the roots in a + bi form. Sketch the roots in the complex plane. (LT: 7a, 7e)

a) The fourth roots of 1.

$1 = 1\,\text{cis}(0 + 2k\pi)$

$1^{\frac{1}{4}} = 1^{\frac{1}{4}}\,\text{cis}\left(\frac{0+2k\pi}{4}\right)$

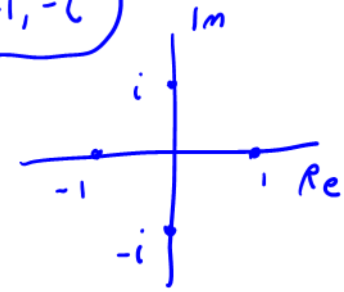
$\cos 0 + i\sin 0 = 1$

$\cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$

$\cos\pi + i\sin\pi = -1$

$\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = -i$

$\boxed{1, i, -1, -i}$



b) The cube roots of 27i.

$27i = 27\,\text{cis}\left(\frac{\pi}{2} + 2k\pi\right)$

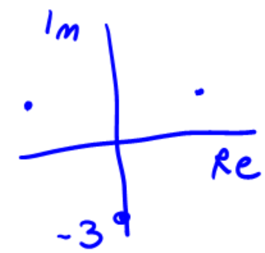
$(27i)^{\frac{1}{3}} = 27^{\frac{1}{3}}\,\text{cis}\left(\frac{1}{3}\left(\frac{\pi}{2} + 2k\pi\right)\right)$

$= 3\,\text{cis}\left(\frac{\pi}{6} + \frac{4k\pi}{6}\right)$

$3\,\text{cis}\frac{\pi}{6} = \frac{3\sqrt{3}}{2} + \frac{1}{2}i$

$3\,\text{cis}\frac{5\pi}{6} = -\frac{3\sqrt{3}}{2} + \frac{1}{2}i$

$3\,\text{cis}\frac{3\pi}{2} = -3i$



c) The cube roots of $\sqrt{2} + \sqrt{2}i$.

$\sqrt{2} + \sqrt{2}i = 2\,\text{cis}\left(\frac{\pi}{4} + 2k\pi\right)$

$= 2\,\text{cis}\left(\frac{\pi + 8k\pi}{4}\right)$

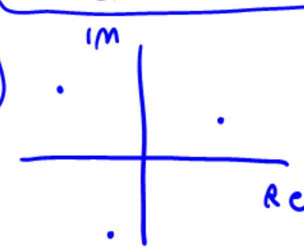
$(\sqrt{2} + \sqrt{2}i)^{\frac{1}{3}} = \sqrt[3]{2}\,\text{cis}\left(\frac{\pi + 8k\pi}{12}\right)$

$\boxed{\sqrt[3]{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)}$

$\boxed{\sqrt[3]{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)} = \boxed{(\sqrt[3]{2})\frac{\sqrt{2}}{2} + \left(\frac{\sqrt[3]{2}\sqrt{2}}{2}\right)i}$

$2^{-\frac{1}{6}} + i\left(2^{-\frac{1}{6}}\right)$

$\boxed{\sqrt[3]{2}\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right)}$



7) Write in the form a + bi. (LT: 7c)

a) $2e^{i\frac{\pi}{4}} = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$= \boxed{\sqrt{2} + \sqrt{2}i}$

b) $e^{\left(\ln 4 + i\frac{\pi}{2}\right)} = e^{\ln 4} e^{i\frac{\pi}{2}} = 4e^{i\frac{\pi}{2}}$

$= \boxed{4i}$

c) $e^{\left(2 - i\frac{\pi}{3}\right)t} = e^{2t} e^{-i\frac{\pi}{3}t}$

$= e^{2t}\cos\left(-\frac{\pi}{3}t\right) + e^{2t}\sin\left(-\frac{\pi}{3}t\right)i$

$= \boxed{e^{2t}\cos\frac{\pi}{3}t - e^{2t}\sin\left(\frac{\pi}{3}t\right)i}$