Name: Solutions

Worksheet 10.3a – Polar Coordinates

1) Convert from rectangular to polar coordinates. (LT: 6a)

a) $(1,0)$	$(1, 0)$
b) $(3,\sqrt{3})$	$\left(2\sqrt{3}, \frac{\pi}{6}\right)$
c) $(-2,2)$	$\left(2\sqrt{2}, \frac{3\pi}{4}\right)$
d) $(-1,\sqrt{3})$	$\left(2, \frac{2\pi}{3}\right)$

$r=\sqrt{9+3}$

$\tan\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

$r=\sqrt{1+3}=2$

$\tan\theta = \frac{\sqrt{3}}{-1}$

2) Convert from polar to rectangular coordinates. (LT: 6a)

a) $\left(3,\frac{\pi}{6}\right)$	$\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$
b) $\left(-6,\frac{3\pi}{4}\right)$	$\left(3\sqrt{2}, -3\sqrt{2}\right)$
c) $\left(0,\frac{\pi}{5}\right)$	$(0, 0)$
d) $\left(5,-\frac{\pi}{2}\right)$	$(0, -5)$

3) Convert the polar equation to an equation in rectangular coordinates. (LT: 6a)

a) $r = 7$	$x^2+y^2=49$
b) $r = 2sin\theta$	$x^2+y^2=2y$
c) $r = \frac{1}{cos\theta - sin\theta}$	$x-y=1$

a) $r^2=49$

$x^2+y^2=49$

b) $r^2=2r\sin\theta$

$x^2+y^2=2y$

c) $r\cos\theta - r\sin\theta = 1$

$x-y=1$

4) Convert the rectangular equation to an equation in polar coordinates. (LT: 6a)

a) $x^2 + y^2 = 25$	$r=5$
b) $x = 5$	$r\cos\theta=5$
c) $y = x^2$	$r\sin\theta = r^2\cos^2\theta$

a) $r^2=25$

$r=5$

b) $r\cos\theta=5$

c) $r\sin\theta = r^2\cos^2\theta$

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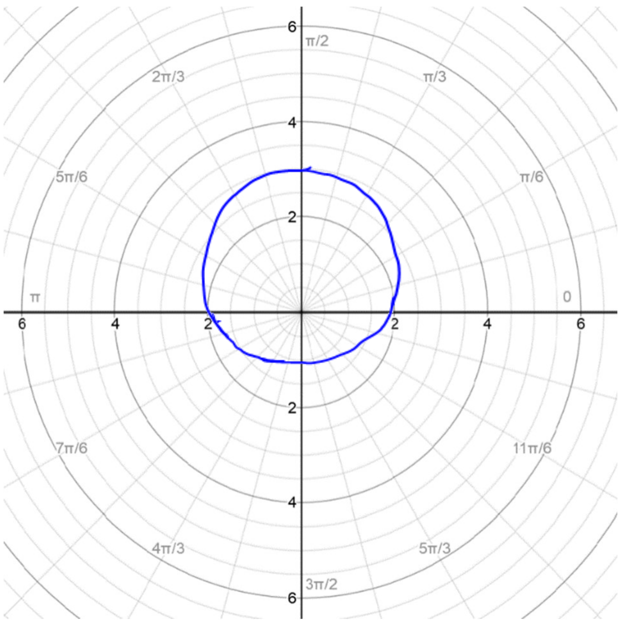
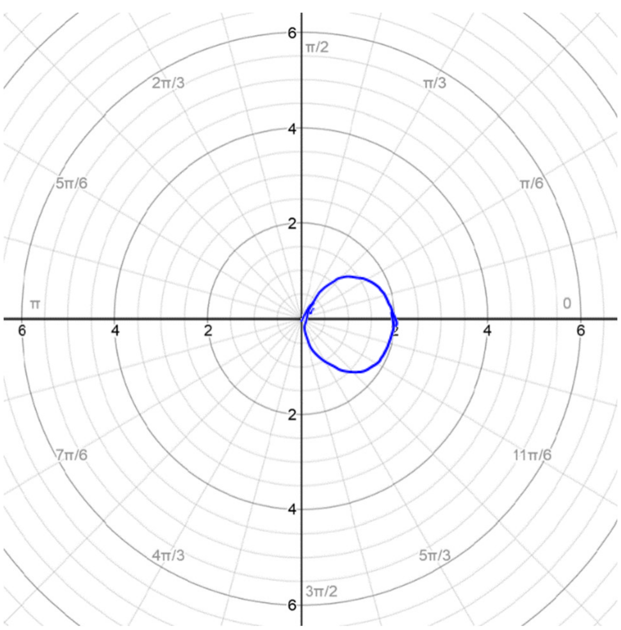
Worksheet 10.3b – Polar Coordinates

Sketch a graph of the polar curve on the grid provided. (LT: 6b)

$r = 2 + sin\theta$

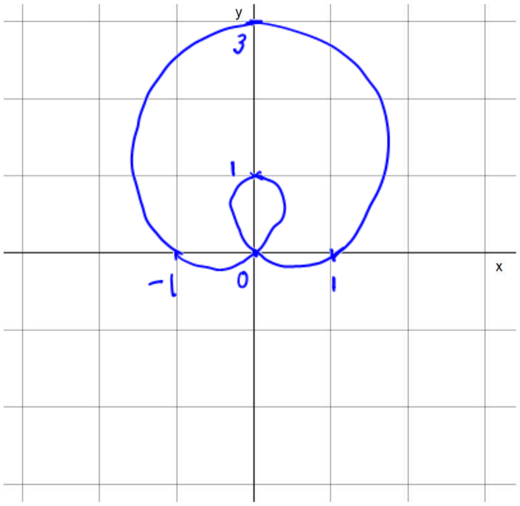


$r = 2cos\theta$

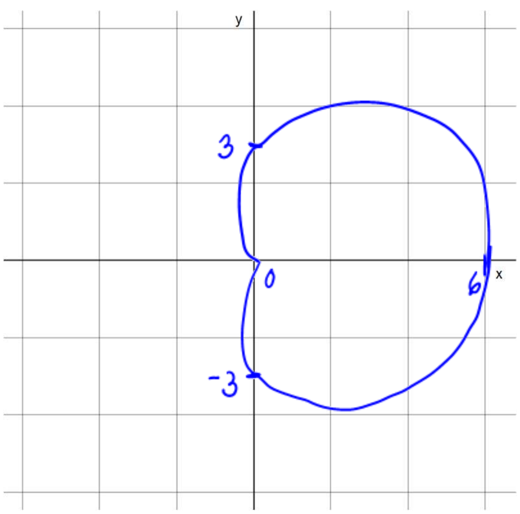


Sketch a graph of the polar curve in the xy-plane. Label all intercepts. (LT: 6b)

$$r = 1 + 2sin\theta$$



$$r = 3 + 3cos\theta$$



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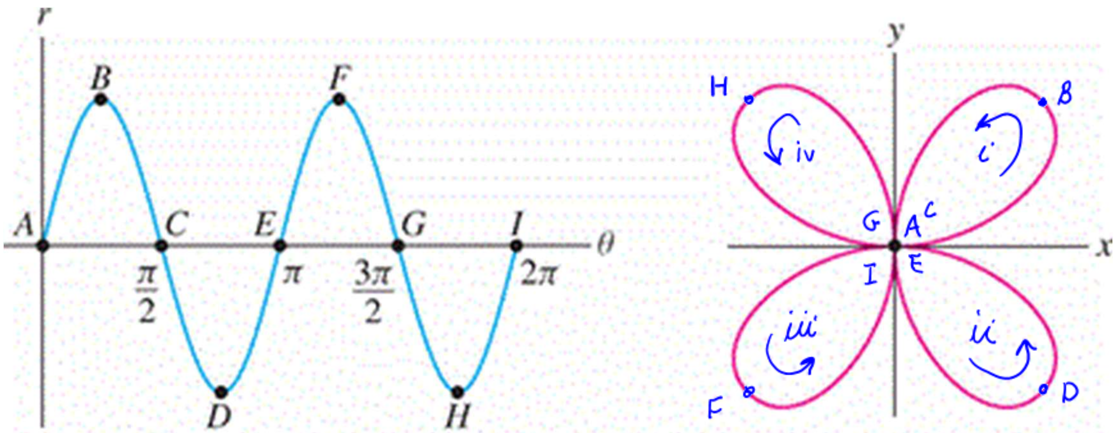
Worksheet 10.3c – Polar Coordinates

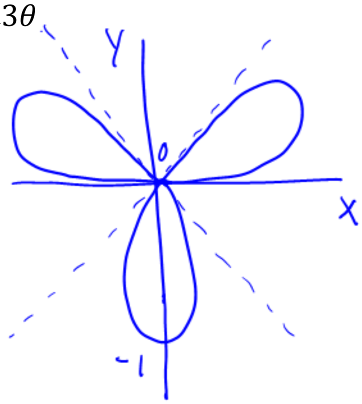
1) The figures below show the graphs of $r = sin2\theta$ in rectangular coordinates and in polar coordinates where it is a rose with four petals. Identify:
   a) The points in (II) that correspond to points A-H in (I).
   b) The parts of the curve in (II) that correspond to the angle intervals
   $i)\ \left[0, \frac{\pi}{2}\right],\quad ii)\ \left[\frac{\pi}{2}, \pi\right],\quad iii)\ \left[\pi, \frac{3\pi}{2}\right],\quad iv)\ \left[\frac{3\pi}{2}, 2\pi\right].$
   (LT: 6b)



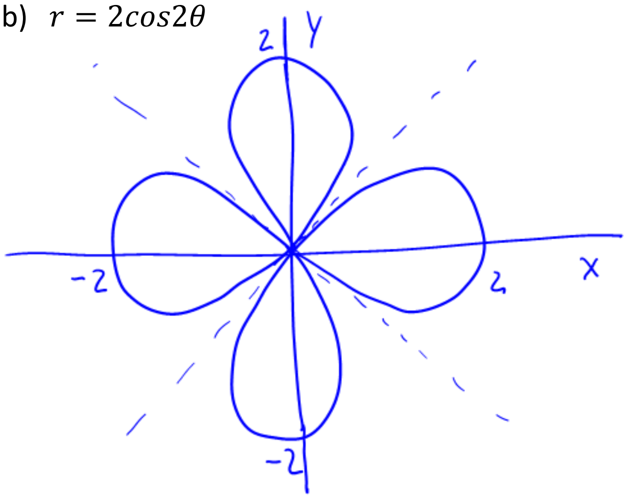
(I) Graph of r as a function of θ, where $r = \sin 2\theta$.

(II) Graph of $r = \sin 2\theta$ in polar coordinates.

2) Sketch a graph of the polar curve. Label all intercepts. (LT: 6b)
   a) $r = sin3\theta$



   b) $r = 2cos2\theta$



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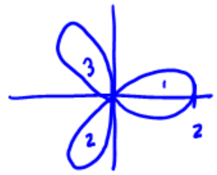
Worksheet 10.4a – Areas and Lengths in Polar Coordinates

1) Set up an integral to compute the length of one loop of the polar curve, $r = 2cos3\theta$. (LT: 6e)

$$\ell = \int_\alpha^\beta \sqrt{r^2 + (dr/d\theta)^2}\, d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{4\cos^2 3\theta + (-6\sin 3\theta)^2}\, d\theta$$

(This is the length of loop 2.)



$2\cos 3\theta = 0$

$\cos 3\theta = 0$

$3\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = \frac{\pi}{6}, \frac{3\pi}{6}$

$$\boxed{\ell = \int_{\frac{\pi}{6}}^{\pi/2} \sqrt{4\cos^2 3\theta + 36\sin^2 3\theta}\, d\theta}$$

other options for the bounds in 1) and 2):

$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$ $\quad 2\int_0^{\frac{\pi}{6}}$ $\quad \frac{1}{3}\int_0^{\pi}$

2) Find the area of one loop of the polar curve, $r = 2cos3\theta$. (LT: 6c)

$$A = \int_\alpha^\beta \frac{1}{2} r^2 d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}(4\cos^2 3\theta)\, d\theta$$

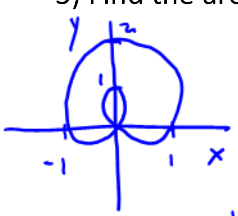
$$= 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}(1 + \cos 6\theta)\, d\theta$$

$$= \left(\theta + \frac{1}{6}\sin 6\theta\right)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$

$$\boxed{\frac{\pi}{3}}$$

3) Find the area of the inner loop of the limacon with polar equation, $r = 1 + 2sin\theta$. (LT: 6c)



$1 + 2\sin\theta = 0$

$\sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$$A = \int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} \frac{1}{2}(1 + 2\sin\theta)^2 d\theta$$

$$= \int_{\frac{7\pi}{6}}^{\frac{3\pi}{2}} (1 + 4\sin\theta + 4\sin^2\theta)\, d\theta$$

$$= \int_{\frac{7\pi}{6}}^{\frac{3\pi}{2}} (1 + 4\sin\theta + 2(1 - \cos 2\theta))\, d\theta$$

$$= \int_{\frac{7\pi}{6}}^{\frac{3\pi}{2}} (3 + 4\sin\theta - 2\cos 2\theta)\, d\theta$$

$$= (3\theta - 4\cos\theta - \sin 2\theta)\Big|_{\frac{7\pi}{6}}^{\frac{3\pi}{2}}$$

$$= \left(\frac{9\pi}{2} - 0 - 0\right) - \left(\frac{21\pi}{6} - 4\cos\frac{7\pi}{6} - \sin\frac{7\pi}{3}\right)$$

$$= \frac{9\pi}{2} - \frac{21\pi}{6} - 2\sqrt{3} + \frac{\sqrt{3}}{2}$$

$$= \pi - \frac{3\sqrt{3}}{2}$$

$$\boxed{\pi - \frac{3\sqrt{3}}{2}}$$

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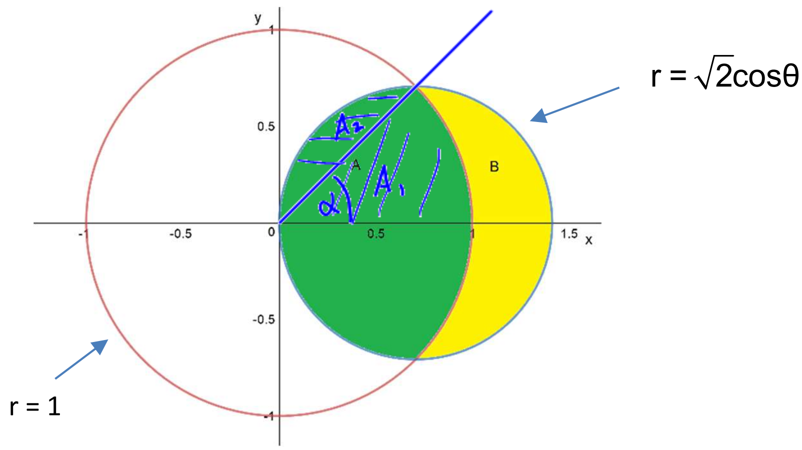
Worksheet 10.4b – Areas and Lengths in Polar Coordinates

1) Consider the figure below.



Intersection: $\sqrt{2}\cos\alpha = 1$

$\cos\alpha = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\alpha = \frac{\pi}{4}$

$A_1 = \int_0^{\frac{\pi}{4}} \frac{1}{2}(1^2)\,d\theta$

$= \frac{1}{2}\theta \Big|_0^{\frac{\pi}{4}}$

$= \frac{\pi}{8}$

$A_2 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2}(\sqrt{2}\cos\theta)^2\,d\theta$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2}(1+\cos 2\theta)\,d\theta$

$= \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \frac{1}{2}\left(\frac{\pi}{2} + 0 - \frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{\pi}{8} - \frac{1}{4}$

a) Find the area of the green shaded region A. (LT: 6d)

$A_A = 2(A_1 + A_2)$

$= 2\left(\frac{\pi}{8} + \frac{\pi}{8} - \frac{1}{4}\right) = \frac{\pi}{2} - \frac{1}{2}$

$\boxed{\frac{\pi}{2} - \frac{1}{2}}$

b) Find the area of the yellow shaded region B. (LT: 6d)

$A_B = 2\int_0^{\frac{\pi}{4}} \frac{1}{2}\left((\sqrt{2}\cos\theta)^2 - 1^2\right)d\theta$

$= \int_0^{\frac{\pi}{4}} (2\cos^2\theta - 1)\,d\theta$

$= \int_0^{\frac{\pi}{4}} (1 + \cos 2\theta - 1)\,d\theta$

$= \frac{1}{2}\sin 2\theta \Big|_0^{\frac{\pi}{4}} = \frac{1}{2}$

OR

A_B = Area of blue circle $- A_A$

$= \pi\left(\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{\pi}{2} - \frac{1}{2}\right)$

$= \frac{1}{2}$

$\boxed{\frac{1}{2}}$

2) Supplement: Find the area of the rectangular stage shown that lies within the pickup region of the microphone which is located at the origin. The pickup region is the region within the polar curve given by the equation, $r = 8 + 8sin\theta$. (LT: 6d)

intersection: $(8+8\sin\alpha)\sin\alpha = 6$

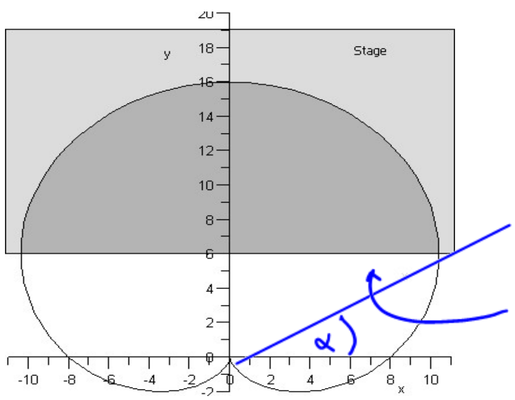
$8\sin\alpha + 8\sin^2\alpha = 6$

$4\sin^2\alpha + 4\sin\alpha - 3 = 0$

$\sin\alpha = \dfrac{-4 \pm \sqrt{16-4(4)(-3)}}{2\cdot 4}$

$\sin\alpha = \dfrac{-4 \pm 8}{8} = -\frac{1}{2} \pm 1$

$\cancel{\sin\alpha = -\frac{3}{2}}$ or $\sin\alpha = \frac{1}{2} \rightarrow \alpha = \frac{\pi}{6}$



$y = 6$

$r\sin\theta = 6$

$r = 6\csc\theta$

$$A = 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}\left[(8+8\sin\theta)^2 - (6\csc\theta)^2\right] d\theta$$

$$2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}\left(64 + 128\sin\theta + 64\sin^2\theta - 36\csc^2\theta\right) d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(64 + 128\sin\theta + 32(1-\cos 2\theta) - 36\csc^2\theta\right) d\theta$$

$$= \left(64\theta - 128\cos\theta + 32\left(\theta - \tfrac{1}{2}\sin 2\theta\right) + 36\cot\theta\right)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \left[32\pi - 0 + 32\left(\tfrac{\pi}{2} - 0\right) + 0\right] - \left[\frac{64\pi}{6} - \frac{128\sqrt{3}}{2} + 32\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) + 36\sqrt{3}\right]$$

$$= 48\pi - \left[\frac{96\pi}{6} - 36\sqrt{3}\right]$$

$$A = 32\pi + 36\sqrt{3}$$

$$\boxed{32\pi + 36\sqrt{3}}$$