Name: Solutions

Worksheet 11.6 – The Ratio and Root Tests

Formally show whether the following series are Absolutely Convergent (AC), Conditionally Convergent (CC), or Divergent (D). (LT: 4b)

1. $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^{100}}$

$a_n = \frac{(-2)^n}{n^{100}}$

$\left|\frac{a_{n+1}}{a_n}\right| = \frac{2^{n+1}}{(n+1)^{100}} \cdot \frac{n^{100}}{2^n}$

$= \frac{2^{n+1}}{2^n} \cdot \frac{n^{100}}{(n+1)^{100}}$

$= 2\left(\frac{n}{n+1}\right)^{100}$

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 2 > 1$

$\sum a_n$ is $\boxed{\text{divergent}}$ by Ratio Test

2. $\sum_{n=0}^{\infty} \left(\frac{5n}{10n+4}\right)^n$

$a_n = \left(\frac{5n}{10n+4}\right)^n$

$\sqrt[n]{|a_n|} = \frac{5n}{10n+4}$

$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \frac{5}{10} = \frac{1}{2} < 1$

$\sum a_n$ is $\boxed{AC}$ by Root Test

3. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{3^n}$

$$a_n = \frac{\sqrt{n}}{3^n}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\sqrt{n+1}}{3^{n+1}} \cdot \frac{3^n}{\sqrt{n}}$$

$$= \sqrt{\frac{n+1}{n}} \cdot \frac{3^n}{3^{n+1}}$$

$$= \sqrt{\frac{n+1}{n}} \left(\frac{1}{3}\right)$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{3} < 1$$

$\sum a_n$ is $\boxed{AC}$ by Ratio Test

4. $\sum_{n=1}^{\infty} (-1)^n \frac{e^n}{n!}$

$$a_n = (-1)^n \frac{e^n}{n!}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{e^{n+1}}{(n+1)!} \cdot \frac{n!}{e^n}$$

$$= \frac{n!}{(n+1)!} \cdot \frac{e^{n+1}}{e^n}$$

$$= \frac{1}{n+1} (e)$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0 < 1$$

$\sum a_n$ is $\boxed{AC}$ by Ratio Test

5. $\sum_{n=1}^{\infty} \frac{1}{(2n)!}$

$$a_n = \frac{1}{(2n)!}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{(2(n+1))!} \cdot \frac{(2n)!}{1}$$

$$= \frac{(2n)!}{(2n+2)!}$$

$$= \frac{1 \cdot 2 \cdot 3 \cdots 2n}{1 \cdot 2 \cdot 3 \cdots (2n)(2n+1)(2n+2)}$$

$$= \frac{1}{(2n+1)(2n+2)}$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0 < 1$$

$\sum a_n$ is $\boxed{AC}$ by Ratio Test

Name Solutions

Worksheet 11.7a Strategy for Testing Series (LT: 4b)

Limits And Convergence Practice Problems

Legend:
C Convergent
AC Absolutely Convergent
CC Conditionally Convergent
D Divergent

Fill out the following chart. For all limits, specify ∞, - ∞, or DNE (does not exist) where appropriate. (LT: 4b)

a_n	$\lim_{n\to\infty} a_n$	$\{a_n\}$ C or D	$\lim_{n\to\infty}\left[(-1)^n a_n\right]$	$\{(-1)^n a_n\}$ C or D	$\sum a_n$ AC, CC, or D	$\sum (-1)^n a_n$ AC, CC, or D
$\frac{1}{n+2}$	0	C	0	C	D	CC
$\frac{n}{n+2}$	1	C	DNE	D	D	D
$\frac{1}{n^2+2}$	0	C	0	C	AC	AC
$\frac{4}{2^n}$	0	C	0	C	AC	AC
$\frac{4n}{2^n}$	0	C	0	C	AC	AC
$\frac{4n!}{2^n}$	∞	D	DNE	D	D	D
$\frac{(n+2)3^n}{n!}$	0	C	0	C	AC	AC
$\frac{4^n}{(3n)^n}$	0	C	0	C	AC	AC

Name: Solutions

Worksheet 11.8 – Power Series

Find the radius and interval of convergence. (LT: 4c)

1) $\sum_{n=1}^{\infty} \frac{x^n}{n^2 3^n}$

$a_n = \frac{x^n}{n^2 3^n}$

$\left|\frac{a_{n+1}}{a_n}\right| = \frac{|x|^{n+1}}{(n+1)^2 3^{n+1}} \cdot \frac{n^2 3^n}{|x|^n}$

$= \left(\frac{n^2}{(n+1)^2}\right) \frac{3^n}{3^{n+1}} \frac{|x|^{n+1}}{|x|^n}$

$= \left(\frac{n}{n+1}\right)^2 \left(\frac{1}{3}\right)|x|$

$\lim_{n \to \infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{3}|x|$

convergent for $\frac{1}{3}|x| < 1$

$|x| < 3$

$-3 < x < 3$

check endpoints:

$x = -3$: $\sum \frac{(-1)^n}{n^2}$ C

$x = 3$: $\sum \frac{1}{n^2}$ C

Interval: $[-3, 3]$

R = 3

2) $\sum_{n=1}^{\infty} \frac{x^n}{n 3^n}$

$a_n = \frac{x^n}{n 3^n}$

$\left|\frac{a_{n+1}}{a_n}\right| = \frac{|x|^{n+1}}{(n+1)3^{n+1}} \cdot \frac{n 3^n}{|x|^n}$

$= \frac{n}{n+1}\left(\frac{3^n}{3^{n+1}}\right)\left(\frac{|x|^{n+1}}{|x|^n}\right)$

$= \frac{n}{n+1}\left(\frac{1}{3}\right)|x|$

$\lim_{n \to \infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{|x|}{3}$

convergent for $\frac{|x|}{3} < 1$

$|x| < 3$

$-3 < x < 3$

check endpoints

$x = -3$: $\sum \frac{(-1)^n}{n}$ C

$x = 3$: $\sum \frac{1}{n}$ D

Interval: $[-3, 3)$

R = 3

3) $\sum_{n=1}^{\infty} \frac{x^n}{3^n}$

Can use geometric series test

Convergent for $|r| = \left|\frac{x}{3}\right| < 1$

divergent otherwise

$\left|\frac{x}{3}\right| < 1$

$|x| < 3$

$(-3, 3)$

No need to check endpoints

Interval: $(-3, 3)$

R = 3

4) $\sum_{n=0}^{\infty} \frac{(-1)^n (x+3)^n}{n!}$

$$a_n = \frac{(-1)^n (x+3)^n}{n!}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{|x+3|^{n+1}}{(n+1)!} \cdot \frac{n!}{|x+3|^n}$$

$$= \frac{n!}{(n+1)!} \frac{|x+3|^{n+1}}{|x+3|^n}$$

$$= \frac{1}{n+1} |x+3|$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0 < 1 \text{ for all } x$$

Interval: $(-\infty, \infty)$

R = ∞

5) $\sum_{n=1}^{\infty} \frac{(-1)^n (x-7)^n}{n}$

$$a_n = \frac{(-1)^n (x-7)^n}{n}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{|x-7|^{n+1}}{n+1} \cdot \frac{n}{|x-7|^n}$$

$$= \frac{n}{n+1} \frac{|x-7|^{n+1}}{|x-7|^n}$$

$$= \frac{n}{n+1} |x-7|$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = |x-7|$$

convergent for $|x-7|<1$

check endpoints

$x-7=-1$

$x=6$

$\sum \frac{(-1)^n(-1)^n}{n}$

D

$x-7=1$

$x=8$

$\sum \frac{(-1)^n 1^n}{n}$

C

Interval: $(6, 8]$

R = 1

6) $\sum_{n=12}^{\infty} n^n (x-2)^n$

use Root Test

$$a_n = n^n (x-2)^n$$

$$\sqrt[n]{|a_n|} = n|x-2|$$

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \infty > 1 \text{ except for } x=2$$

Interval: $x=2$

R = 0