Name: Solutions

Worksheet 11.1 - Sequences

1) Match each sequence with its general term by writing the appropriate number (i, iii, iii, or iv) next to the sequence. (LT: 4a)

a_1, a_2, a_3, a_4, …		General term	
a) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$	iv	i)	$\cos \pi n$
(b) -1, 1, -1, 1, …	i	ii)	$\frac{n!}{2^n}$
(c) 1, -1, 1, -1, …	iii	iii)	$(-1)^{n+1}$
(d) $\frac{1}{2}, \frac{2}{4}, \frac{6}{8}, \frac{24}{16}, \ldots$	ii	iv)	$\frac{n}{n+1}$

2) Calculate the first four terms of each sequence, starting with n = 1. (LT: 4a)

a) $c_n = \frac{3^n}{n!}$ $\quad 3, \frac{3^2}{2}, \frac{3^3}{6}, \frac{3^4}{24}$

$\{\ 3,\ \frac{9}{2},\ \frac{9}{2},\ \frac{27}{8},\ldots\}$

b) $b_n = 5 + \cos \pi n$

$\{\ 4,\ 6,\ 4,\ 6,\ldots\}$

c) $a_n = \frac{(2n-1)!}{n!}$

$1, \frac{3\cdot 2}{2}, \frac{5\cdot4\cdot3\cdot2}{3\cdot2}, \frac{7\cdot6\cdot5\cdot4\cdot3\cdot2}{4\cdot3\cdot2}$

$\{\ 1,\ 3,\ 20,\ 210,\ldots\}$

3) Find a formula for the nth term of each sequence. (LT: 4a)

a) $\frac{1}{1}, \frac{-1}{8}, \frac{1}{27}, \ldots$

n= 1 2 3

$a_n = \frac{(-1)^{n+1}}{n^3}$

b) $\frac{2}{6}, \frac{3}{7}, \frac{4}{8}, \ldots$

n= 1 2 3

$a_n = \frac{n+1}{n+5}$

4) Determine the limit of the sequence and state whether the sequence converges or diverges (C or D). (LT: 4a)

Sequence	Limit	C or D	Sequence	Limit	C or D
a) $b_n = \dfrac{5n-1}{12n+9}$	$\frac{5}{12}$	C	k) $d_n = \ln 5^n - \ln n!$	$-\infty$	D
b) $b_n = (-1)^n\left(\dfrac{5n-1}{12n+9}\right)$	DNE	D	l) $a_n = \left(2+\dfrac{4}{n^2}\right)^{1/3}$	$2^{1/3}$	C
c) $a_n = \sqrt{4+\dfrac{1}{n}}$	2	C	m) $c_n = \ln\left(\dfrac{2n+1}{3n+4}\right)$	$\ln\left(\frac{2}{3}\right)$	C
d) $a_n = \cos^{-1}\left(\dfrac{n^3}{n^3+1}\right)$	0	C	n) $y_n = \dfrac{e^n}{2^n}$	∞	D
e) $a_n = 10+\left(-\dfrac{1}{9}\right)^n$	10	C	o) $a_n = \dfrac{(\ln n)^2}{n}$	0	C
f) $c_n = 1.01^n$	∞	D	p) $a_n = \dfrac{(-1)^n(\ln n)^2}{n}$	0	C
g) $a_n = 2^{1/n}$	1	C	q) $b_n = \dfrac{3-4^n}{2+7\cdot 4^n}$	$-\frac{1}{7}$	C
h) $c_n = \dfrac{n!}{9^n}$	∞	D	r) $a_n = \left(1+\dfrac{1}{n}\right)^n$	e	C
i) $a_n = \dfrac{3n^2+n+2}{2n^2-3}$	$\frac{3}{2}$	C	s) $a_n = \dfrac{1}{\ln\left(1+\dfrac{1}{n}\right)}$	∞	D
j) $a_n = \dfrac{\cos n}{n}$	0	C	t) $a_n = n\sin\dfrac{\pi}{n}$	π	C

r) $y = \left(1+\frac{1}{x}\right)^x$

$\ln y = x\ln\left(1+\frac{1}{x}\right)$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}$$

$$\stackrel{H}{=} \lim_{x\to\infty} \frac{\frac{x}{x+1}\left(-\frac{1}{x^2}\right)}{\left(-\frac{1}{x^2}\right)}$$

$$= 1$$

$\lim_{x\to\infty} \ln y = 1$

$\lim_{x\to\infty} y = e^1 = e \rightarrow \lim_{n\to\infty} \ln\left(1+\frac{1}{n}\right)^n = 1$

t) $\lim_{x\to\infty} x\sin\frac{\pi}{x}$ type $\infty\cdot 0$

$$= \lim_{x\to\infty} \frac{\sin\left(\frac{\pi}{x}\right)}{\left(\frac{1}{x}\right)} \quad \text{type } \frac{0}{0}$$

$$\stackrel{H}{=} \lim_{x\to\infty} \frac{\cos\left(\frac{\pi}{x}\right)\left(-\frac{\pi}{x^2}\right)}{\left(-\frac{1}{x^2}\right)}$$

$$= \pi$$

$\lim_{n\to\infty} n\sin\frac{\pi}{n} = \pi$

Name: Solutions

Worksheet 11.2 - Series

1) Find a formula for the general term a_n of the infinite series and write the series in the form $\sum_{n=1}^{\infty} a_n$

$$\frac{1}{1} - \frac{2^2}{2\cdot 1} + \frac{3^3}{3\cdot 2\cdot 1} - \frac{4^4}{4\cdot 3\cdot 2\cdot 1} + \cdots$$

$n \to$ 1 2 3 4

$$\boxed{\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^n}{n!}}$$

2) Circle each geometric series.

a) $\sum_{n=0}^{\infty} \frac{7^n}{29^n}$

b) $\sum_{n=3}^{\infty} \frac{1}{n^4}$

c) $\sum_{n=0}^{\infty} \frac{n^2}{2^n}$

d) $\sum_{n=5}^{\infty} \pi^{-n}$

3) (LT: 4b)

Formally show whether the following series are convergent or divergent.	If the series is a convergent geometric series, enter the sum.
a) $\sum_{n=-4}^{\infty} \left(-\frac{4}{9}\right)^n$ is a convergent geometric series. $\lvert r\rvert = \frac{4}{9} < 1$ Sum: $\frac{\left(-\frac{4}{9}\right)^{-4}}{1-\left(-\frac{4}{9}\right)} = \frac{9^4}{4^4\left(\frac{13}{9}\right)}$	$\frac{9^5}{13\cdot 4^4}$
b) $\sum_{n=0}^{\infty} e^{3-2n}$ is a convergent geometric series $\lvert r\rvert = e^{-2} < 1$ Sum: $\frac{e^3}{1-e^{-2}} = \frac{e^5}{e^2-1}$	$\frac{e^5}{e^2-1}$
c) $\sum_{n=1}^{\infty} \left(\frac{n^2}{5n^2+4}\right)$ diverges by the DT $\lim_{n\to\infty} \frac{n^2}{5n^2+4} = \frac{1}{5} \neq 0$	

4) Suppose that $s = \sum_{n=1}^{\infty} a_n$ is an infinite series with partial sum $s_n = 5 - \frac{2}{n^2}$.

a) What is the value of a_3?

$$a_3 = S_3 - S_2 = 5 - \frac{2}{3^2} - \left(5 - \frac{2}{2^2}\right)$$
$$= \frac{1}{2} - \frac{2}{9} = \frac{9-4}{18}$$

$\boxed{\frac{5}{18}}$

b) Find a general formula for a_n for $n > 1$.

$$a_n = S_n - S_{n-1} = \left(5 - \frac{2}{n^2}\right) - \left(5 - \frac{2}{(n-1)^2}\right)$$
$$= \frac{2}{(n-1)^2} - \frac{2}{n^2} = \frac{2n^2 - 2(n-1)^2}{n^2(n-1)^2} = \frac{2n^2 - 2n^2 + 4n - 2}{n^2(n-1)^2}$$

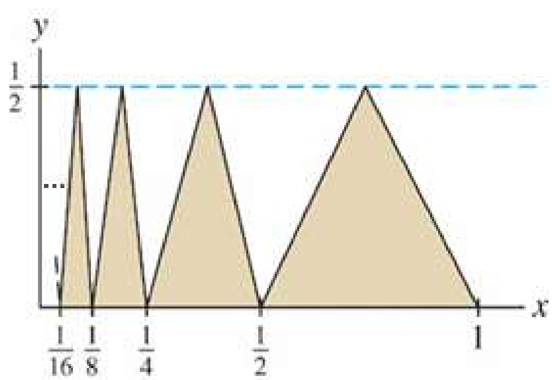
$\boxed{\frac{2(2n-1)}{n^2(n-1)^2}}$

c) Find the sum, $s = \sum_{n=1}^{\infty} a_n$.

$$S = \lim_{n\to\infty} S_n = 5$$

$\boxed{5}$

5) Compute the total area of the (infinitely many) triangles in the figure. (LT: 4b)



$$A = \frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{4}\right)\left(\frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{8}\right)\left(\frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{16}\right)\left(\frac{1}{2}\right) + \cdots$$
$$= \sum_{n=1}^{\infty} \frac{1}{8}\left(\frac{1}{2}\right)^{n-1}$$
$$= \frac{\frac{1}{8}}{1 - \frac{1}{2}}$$
$$= \frac{1}{4}$$

$\boxed{\frac{1}{4}}$