# W01 - Examples

## Volume using cylindrical shells

### 01 - Revolution of a triangle

A rotation-symmetric 3D body has cross section given by the region between $y = 3x + 2$, $y = 6 - x$, $x = 0$, and is rotated around the $y$-axis. Find the volume of this 3D body.

#### Solution

1. Define the cross section region.
    - Bounded above-right by $y = 6 - x$.
    - Bounded below-right by $y = 3x + 2$.
    - These intersect at $x = 1$.
    - Bounded at left by $x = 0$.
2. Define range of integration variable.
    - Rotated around $y$-axis, therefore use $x$ for integration variable (shells!).
    - Integral over $x \in [0, 1]$:

$$V = \int_0^1 2\pi Rh \, dr$$

3. Interpret $R$.
    - Radius of shell-cylinder equals distance along $x$:

$$R(x) = x$$

4. Interpret $h$.
    - Height of shell-cylinder equals distance from lower to upper bounding lines:

$$\begin{aligned} h(x) &= (6 - x) - (3x + 2) \\ &= 4 - 4x \end{aligned}$$

5. Interpret $dr$.
    - $dr$ is limit of $\Delta r$ which equals $\Delta x$ here so $dr = dx$.
6. Plug data in volume formula.
    - Insert data and compute integral:

$$\begin{aligned} V &= \int_0^1 2\pi Rh \, dr \\ &= \int_0^1 2\pi \cdot x(4 - 4x) \, dx \\ &= 2\pi \left( 2x^2 - \frac{4x^3}{3} \right) \Bigg|_0^1 = \frac{4\pi}{3} \end{aligned}$$

### 02 - Revolution of a sinusoid

Consider the region given by revolving the first hump of $y = \sin(x)$ about the $y$-axis. Set up an integral that gives the volume of this region using the method of shells.

Solution

# Integration by parts

## 03 - A and T factors

Compute the integral: $\int x \cos x \, dx$

### Solution

1. Choose $u = x$.
    - Set $u(x) = x$ because $x$ *simplifies* when differentiated.
      (By the trick: $x$ is *Algebraic*, i.e. more "$u$", and $\cos x$ is *Trig*, more "$v$".)
    - Remaining factor must be $v'$:
      $$v'(x) = \cos x$$
2. Compute $u'$ and $v$.
    - Derive $u$:
      $$u' = 1$$
    - Antiderive $v'$:
      $$v = \sin x$$
    - Obtain chart:
      $$\begin{array}{c|c} u = x & v' = \cos x \\ \hline u' = 1 & v = \sin x \end{array} \quad \begin{array}{ccl} \longrightarrow & \int u \cdot v' & \quad \text{original} \\ \longrightarrow & \int u' \cdot v & \quad \text{final} \end{array}$$
3. Plug into IBP formula.
    - Plug in all data:
      $$\int x \cos x \, dx = x \sin x - \int 1 \cdot \sin x \, dx$$
    - Compute integral on RHS:
      $$\int x \cos x \, dx = x \sin x + \cos x + C$$

      Note: the *point* of IBP is that this integral is easier than the first one!
4. Final answer is: $x \sin x + \cos x + C$

## 04 - Hidden A

Compute the integral:

$$\int \ln x \, dx$$

Solution