Unit 04 - Essential problems

Parametric curves

Convert parametric curve to function graph

Write the following curves as the graphs of a function $y = f(x)$. (Find $f(x)$ for each case.)

(a) $x = t + 3$, $y = 4t$ and $0 < t < 1$

(b) $x = \cos t$, $y = \sin^2 t$ and $0 < t < 2\pi$

Sketch each curve.

Convert function graph to parametric curve

Find parametric curves $c(t) = \big(x(t),\, y(t)\big)$ whose images are the following graphs:

(a) $y = 3x - 4$ and $c(0) = (2, 2)$

(b) $y = 3x - 4$ and $c(3) = (2, 2)$

Parametric concavity

Find the intervals of t on which the parametric curve $c(t) = (t^2,\, t^3 - 4t)$ is concave up.

Cycloid - Arclength and surface area of revolution

Consider the cycloid given parametrically by $c(t) = (t - \sin t, 1 - \cos t)$.

(a) Find the length of one arch of the cycloid.

(b) Suppose one arch of the cycloid is revolved around the x-axis. Find the area of this surface of revolution.

Polar curves

Convert points: Cartesian to Polar

Convert the Cartesian (rectangular) coordinates for these points into polar coordinates:

(a) $(1,0)$ (b) $(3,\sqrt{3})$ (c) $(-2,2)$ (d) $(-1,\sqrt{3})$

Polar curve - Vertical or horizontal tangent lines

Find all points on the given curve where the tangent line is horizontal or vertical.

$$r = \cos\theta \qquad \theta \in [0, 2\pi)$$

Hint: First determine parametric Cartesian coordinate functions using θ as the parameter.

Convert equations: Cartesian to Polar

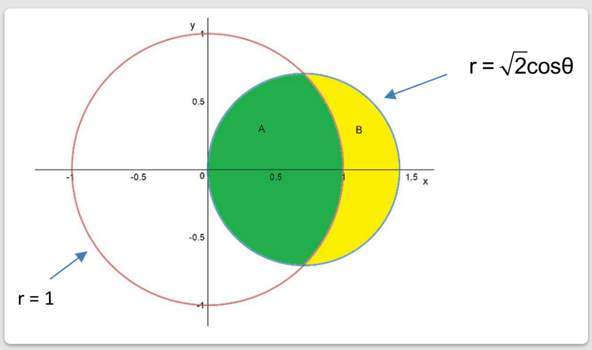
Convert the Cartesian equation to a polar equation. Be sure to simplify.

(a) $x^2 + y^2 = 25$ (b) $x = 5$ (c) $y = x^2$

Polar coordinates - lunar areas

(a) Find the area of the green region.

(b) Find the area of the yellow region.



Area of an inner loop

A limaçon is given as the graph of the polar curve $r = 1 + 2\sin\theta$.

Find the area of the inner loop of this limaçon.

Complex numbers

Complex forms - exponential to Cartesian

Write each number in the form $a + bi$.

(a) $2e^{i\frac{\pi}{4}}$ (b) $e^{\ln 4 + i\frac{\pi}{2}}$

Complex products and quotients using polar

For each pair of complex numbers z and w, compute:

$$zw, \qquad \frac{z}{w}, \qquad \frac{1}{z}$$

(a) $z = 1 + \sqrt{3}i, \quad w = \sqrt{3} + i$

(b) $z = 2\sqrt{3} - 2i, \quad w = 6i$

(Use polar forms with $\theta \in [0, 2\pi)$.)

Complex powers using polar

Using De Moivre's Theorem, write each number in the form $a + bi$.

(a) $(1+i)^{16}$ (b) $(\sqrt{3} - i)^5$

(First convert to polar/exponential, then compute the power, then convert back.)

Complex roots using polar

Find each of the indicated roots.

(a) The four 4^{th} roots of 1.

(b) The three cube (3^{rd}) roots of $\sqrt{2} + \sqrt{2}i$.

Try to write your answer in $a + bi$ form if that is not hard, otherwise leave it in polar form.