Unit 02 - Essential problems

Arc length

Arc length - reversed x and y roles

Find the arc length of the curve that satisfies the equation $x = \frac{1}{12}y^3 + y^{-1}$ over $y \in [1, 2]$.

Surface areas of revolutions - thin bands

Surface area: revolved cubic

The curve $y = x^3$ over $x \in [0, 2]$ is revolved around the *x-axis*.

Find the area of the resulting surface.

Surface area: parabolic reflector

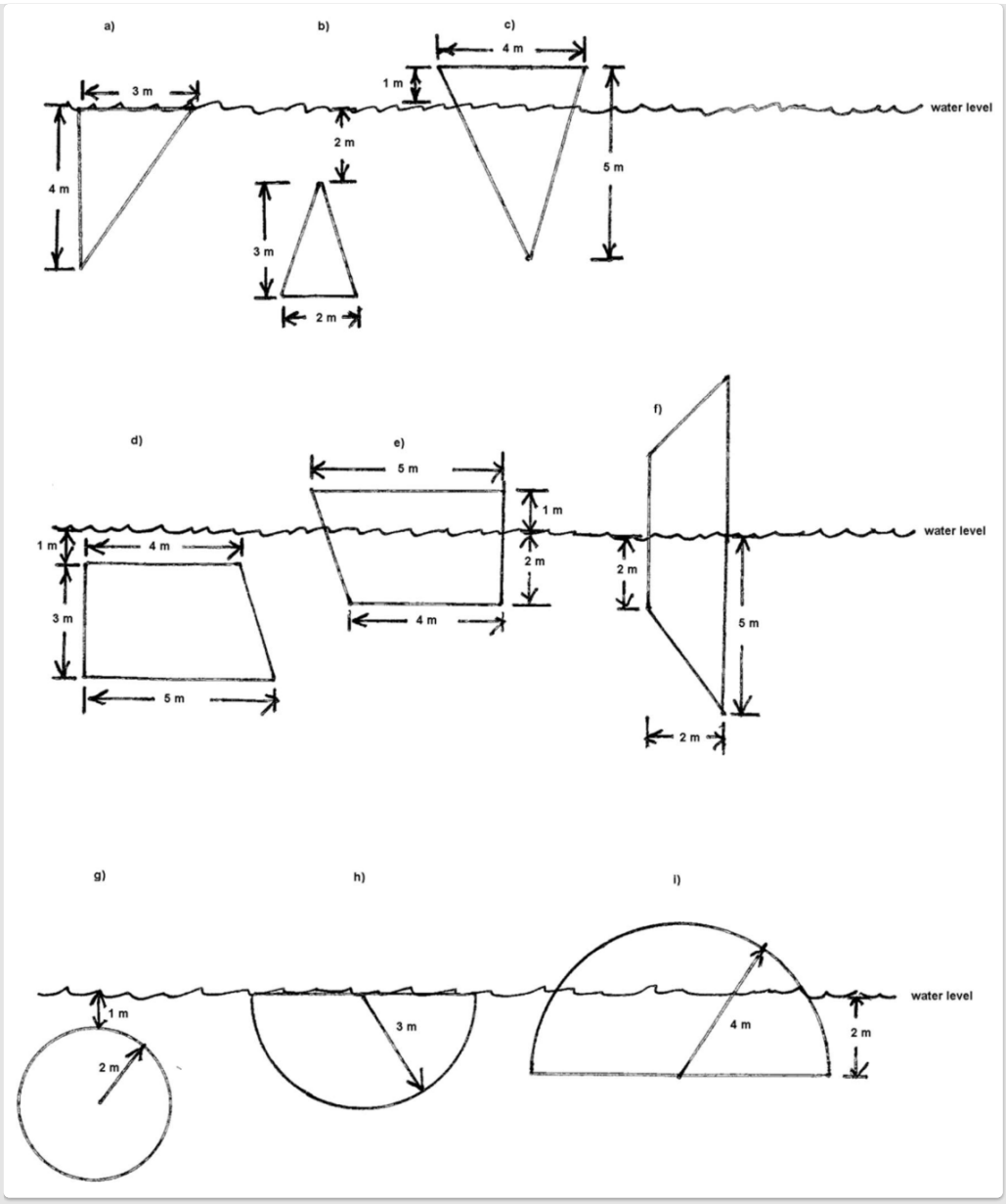
A parabolic reflector is given by rotating the curve $y = x^2$ around the *y-axis* for $x \in [0, 2]$.

What is the surface area of this reflector?

Hydrostatic pressure

Fluid force on various plates

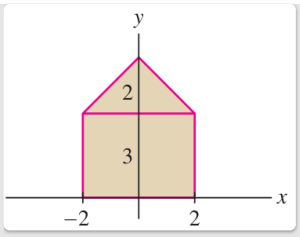
For diagrams (a)-(i) below, set up an integral to compute the hydrostatic force on the plate.



Moments and CoM

Center of mass of a house

A “house” is the region bounded by the (non-regular) pentagon with vertex points at $(0, 5)$, $(2, 3)$, $(2, 0)$, $(-2, 0)$, $(-2, 3)$. Find the CoM of the house using additivity of moments.



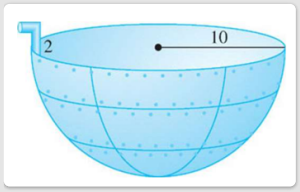
CoM of region between curves

Find the CoM of the region between the graph of $y = e^x$ and the graph of $y = 1$ over $x \in [0, 1]$.

Work performed

Pumping water from hemispherical tank

A hemispherical tank (radius 10m) is full of water. A pipe allows water to be pumped out, but requires pumping up 2m above the top of the tank.



- (a) Set up an integral that computes the total work required to pump all the water out of the tank, assuming it is completely full.
- (b) Now assume the tank start out full just to 6m. What does the integral become?

Building a conical tower

Set up an integral that computes the work done (against gravity) to build a circular cone-shaped tower of height 4m and base radius 1.2m out of a material with mass density 600kg/m^3.

Work to raise a leaky bucket

A bucket of water is raised by a chain to the top of a 50-foot building. The water is leaking out, and the chain is getting lighter.

The bucket weighs $4\,\text{lbs}$, the initial water weighs $30\,\text{lbs}$, and the chain weighs $0.25\,\text{lb/ft}$, and the water is leaking at a rate of $0.3\,\text{lbs/sec}$ as the bucket is lifted at a rate of $3\,\text{ft/sec}$.

What is the total work required to raise the bucket of water?

Improper integrals

Comparison test

Use the comparison test to determine whether the integral converges:

$$\int_1^\infty \frac{dx}{x^4+\sqrt{x}}$$

Computing improper integrals, Part II

For each integral below, give the limit interpretation of improper integral and then compute the limit. Based on that result, state whether the integral converges. If it converges, what is its value?

- (a) $\int_0^1 \ln x\,dx$
- (b) $\int_{-\infty}^{+\infty} xe^{-x^2}\,dx$
- (c) $\int_1^2 \frac{1}{(x-1)^2}\,dx$