Section 6.3: Volumes By Cylindrical Shells

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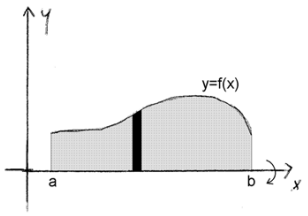
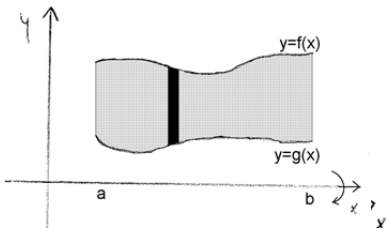
6.3 Volumes By Cylindrical Shells

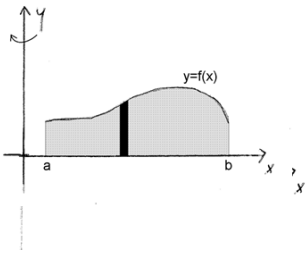
Which Solid Region?

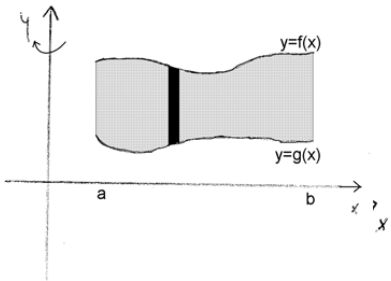




What kind of solid region is generated by revolving the black rectangle around the indicated axis?

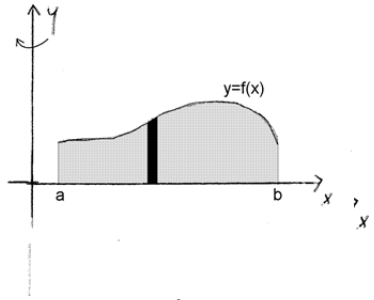
Which Solid Region?





What kind of solid region is generated by revolving the black rectangle around the indicated axis?

Shells,Case A



This rectangle will generate a cylindrical shell.

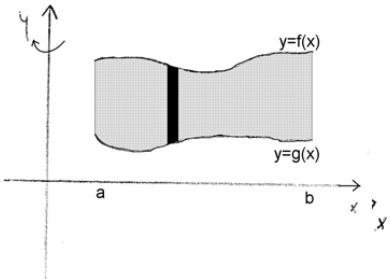
Volume = (circumference)(height)(thickness)= cht

$c = 2\pi r = 2\pi x_i; \qquad h = y = f(x_i); \qquad t = \Delta x$

$V_i \approx 2\pi x_i f(x_i)\Delta x$

$V = \int_a^b 2\pi x f(x) dx$

Shells,Case B



This rectangle will generate a cylindrical shell.

Volume = (circumference)(height)(thickness)= cht

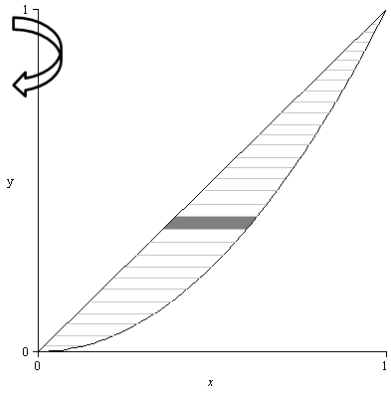
$c = 2\pi r = 2\pi x_i; \qquad h = f(x_i) - g(x_i); \qquad t = \Delta x$

$V_i \approx 2\pi x_i (f(x_i) - g(x_i))\Delta x$

$V = \int_a^b 2\pi x (f(x) - g(x)) dx$

Review

Find the volume of the solid region obtained by revolving the region bounded by the curves, $y = x$ and $y = x^2$, about the y-axis.



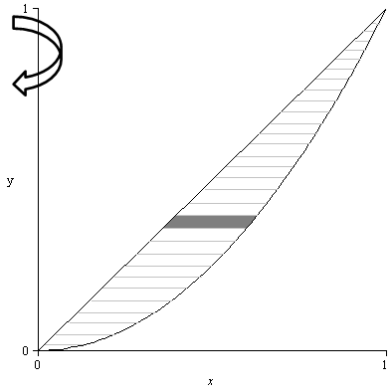
Divide the region into horizontal rectangles.

Focus on one rectangle.

Consider the solid region obtained by revolving the rectangle.

Review - continued

This rectangle generates a washer:



$$V_i \approx \pi((\sqrt{y_i})^2 - {y_i}^2)\Delta y$$

$$V = \int_0^1 \pi((\sqrt{y})^2 - y^2)dy$$

$$= \int_0^1 \pi(y - y^2)dy$$

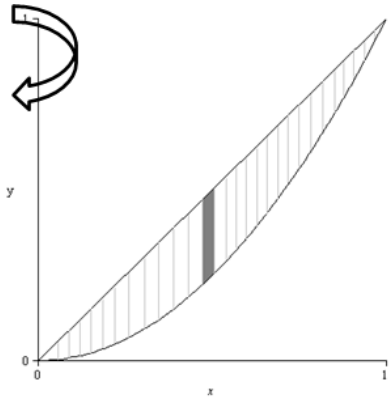
$$= \pi[\frac{y^2}{2} - \frac{y^3}{3}]_0^1$$

$$= \pi[\frac{1}{2} - \frac{1}{3}]$$

$$= \frac{\pi}{6}$$

Volume By Shells

Suppose we divide the plane region into vertical rectangles:



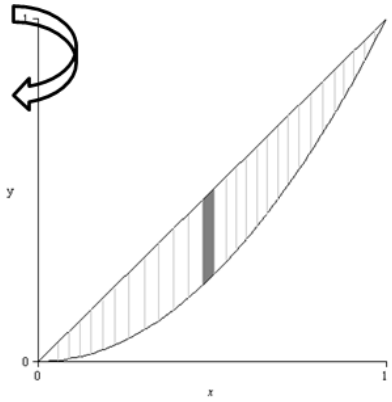
Divide the region into vertical rectangles.

Focus on one rectangle.

Consider the solid region obtained by revolving the rectangle about the appropriate axis.

Volume By Shells - continued

This rectangle generates a cylindrical shell.



$$V_i \approx 2\pi x_i(x_i - {x_i}^2)\Delta x$$

$$V = \int_0^1 2\pi x(x - x^2)dx$$

$$= \int_0^1 2\pi(x^2 - x^3)dx$$

$$= 2\pi[\tfrac{x^3}{3} - \tfrac{x^4}{4}]_0^1$$

$$= 2\pi(\tfrac{1}{3} - \tfrac{1}{4})$$

$$= \tfrac{\pi}{6}$$

Try It

Find the volume of revolution about the y-axis for the region between $x = 0$ and $x = \pi$ that is bounded by the curves, $y = \sin x$ and $y = 0$.