Ad Calculum Situs Constituendum

On the Constitution of a Calculus of Situation

G. W. Leibniz, 1712

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To establish a calculus of situation it is useful to reduce everything from words to symbols, and to continue the matter up until one has an analysis, that is, until the demonstrations of theorems come out without the aid of cleverness, by a sure thread of reasoning.

Let a point be denoted by a capital letter, such as A, or indeed by Greek letters if desired.

Let a locus which contains multiple points, or about which it is not determined whether it contains multiple points, be denoted by the letter denoting a point, with a line drawn above as $\overline{X}$, and any point which is in this locus will be called X. For instance, if A is in $\overline{X}$, some X is A.

Let A be in $\overline{X}$ and in addition let $\mathrm{A}| \simeq |\overline{X}$; then $\overline{X}$ will be a point.

If T is time and to every T there corresponds its own X, then $\overline{X}$ will be a curve. Or $\overline{X}$ will arise from the motion of a point; let there be many X or if $\mathrm{X}| \simeq |\mathrm{X}$ to T and $\overline{T}$ is time, then $\overline{X}$ will be a curve through X to T; it is understood that to each X there is its own T, and to each T its own X. Or X and T are coordinates.

Thus a curve intersects a curve in a point, because a *section* is the common locus of two mobile things at the same instant, so that after that instant there is nothing common to those mobile things. But this definition cannot be applied to a section in the motion of surfaces and bodies, because any two curves generating their own surface and any two surfaces generating their own body are rarely such that they can be congruent in whole or in part. But a point is always congruent to a point, hence to stand for our general definition of section, that it is the whole common to two magnitudes not having a common part.

If two curves do not have a common part, and so the two points describing them cannot have a common locus except in an instant (for a locus beyond an instant is a part of a curve), but the locus in an instant is a point. And they do not have anything in common but a point.

A *straight line* will be most conveniently (best) defined as a curve of which a part is similar to the whole.

Let there be a line $\overline{X}$ and let $\overline{Y}$ be such that $\overline{Y}$ is in $\overline{X}$, or every Y is X, and let $\overline{Y} \sim \overline{X}$; $\overline{X}$ will be called a bounded line. Points do not enter into this definition; further from this it follows that parts of a line are similar to each other, namely since they are similar to the same whole. It also follows that every line can be extended. Indeed, since the extension of a part has given the whole, a consequence is that the whole can also be extended so that it is the part of another whole.

From the nature of similarity it follows that two lines cannot meet each other except in one point. Let them have common point A and let AX and AY go out from it and again come together at B. Let X and Y move with velocities which are as AXB to AYB, so they will meet in point B. But let the motion of each point be uniform. Since $A(X) \simeq AX$ and $A(Y) \simeq AY$ and the motion through $A(X)$ and $A(Y)$ is similar to the motion through AX, AY, and so $A(X)(Y)$ and AXY are determined similarly, we will have also $A(X)(Y) \simeq AXY$. Therefore since X and Y do not coincide, neither will (X) and (Y) coincide, and so neither will it be possible for the point B to exist.

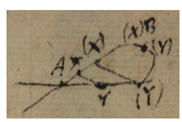

Figure 1

Hence given two points, a line is determined which connects the points; or lines congruent at the extremes are congruent in whole.

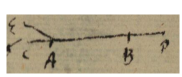

Figure 2

And given two points an infinite line passing through the two points is determined. Indeed, the line AB is determined, and it can only be extended in one way in either direction, as toward C or D, since if it could be extended from A toward C and E, lines EAB and CAB would be given which have more than a point in common.

Two lines AB and CD are similar to each other: Indeed, AB, as well as CD, is determined from the fact that the two points are connected through the line, a part of which is similar to the whole. And so the one determination is similar to the other, for which reason the things determined, the lines AB and CD, are also similar.

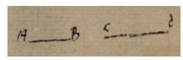

Figure 3

Hence two equal lines are congruent.

A point on a line is unique of its relation to two points on the line. Let there be another point D, which relates in the same way to A and B as C does, therefore it is possible to arrive at D as at C with respect to A and B, by a line through A and B, facing toward [*ad partes*] B it is possible to arrive at D. Contrary to what has been shown about the line.

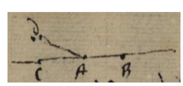

Figure 4

Now points having the same relation are those which can be determined in the same way from the points to which they have the same relation. And if they cannot be determined, neither can they have the same relation.

Hence if $A.B.X$ is unique, then $\overline{X}$ will be a line.

If two lines meet, and it does not matter how large the lines are, the meeting will be called an *angle*.

And so if two lines that meet are congruent to another two lines that meet, provided they are extended far enough, the angle will be said to be *equal*.

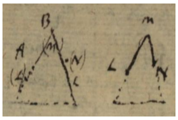

Figure 5

And so if a line AB is given, with the angle ABC given, an indefinite line is given through C, which makes the given angle to AB. From this one understands that if equal angles are determined in the same way, it will happen that if AB is given and a point C is given through which a line from B is to be drawn, the angle is determined.

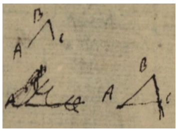

Figure 6

Hence, with two sides AB, AC, of a triangle given in position, the angle to B or to C is given.

But Euclid does not explain from whence he gathers that the angle of something is understood to be greater or less. However, his intention seems to be this, that an angle is considered in a plane and defines a quantity by the part of the plane which it cuts off by the lines BA and BC extended infinitely: Since the part cut off by DBC falls inside the previous one, the angle is a part of the previous one. But then it will be necessary to first explain the nature of the plane.

If two indefinite lines meet, all the lines joining any two points of the two lines fall in a plane and make up a plane. And hence the plane F will be the locus of all points unique with respect to three given points not falling on the same line. Let a point D be taken anywhere on line AC, and a point E on line BC, let the line through DE be drawn, and on it let a point F be taken. I claim that it is unique with respect to the

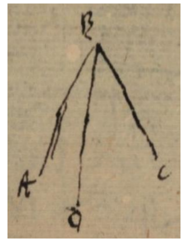

Figure 7

points A, B, C; indeed, D is unique with respect to A and C, and E is unique with respect to B and C, therefore $D.E$ is unique with respect to A, B, C, and what is unique with respect to $D.E$ is unique with respect to A, B, C.

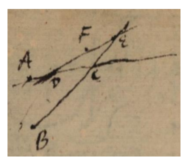

Figure 8

But because it seems redundant for there to be two lines, whatever lines we take joining the points of the lines AC, BC, because † *meets* one point once, we could have taken them by a certain law, CD and CE equal if desired, and thus have joined all DE. But it is best to choose a line from an angle. Since one line relates in the same way to one or the other side. And that will be said to make the previous one at right angles. Thus ABC and CBD are equal, when also ABE and EBD are equal $..ABE.. \simeq ..BDC.. \simeq ..ABE.. \simeq ..DBC$. One should see if it can be shown also that $ABC = ABE$. Let $ABC > ABE$; then $ABC - ABE = DBC - DBE$ and $ABC + DBE = ABE + DBE$; but from here in order to say that angle ABE is equal to the angle ABC there is first need for consideration of the plane, about which more soon. So if we define a right angle as when a one line intersects another in the same way on both sides, one should see if there is no redundancy or theft.

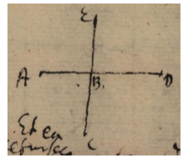

Figure 9

It seems that lines can be said to be meeting in general, to the extent that the meeting relates in the same way and AEB and BEA are congruent. One should see if it can be deduced from this that AEB and CED are also congruent, but the matter will be demonstrated soon after establishing the nature of the plane.

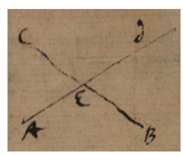

Figure 10

Thus, letting line CD be moved toward line AB, in such a way that it is carried with its same point E to $*(E)*$ through AB, and letting the angle AEC always be equal to CEB, a plane will be described; but one must presuppose that, while moving, the line CD does not gyrate around AB as an axis.

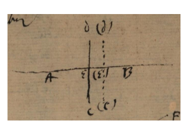

Figure 11

Therefore, for this to be avoided in the construction, one can conceive that the line AF is given and the line CDE is carried over two lines at once; thus it will remain in a plane. But when we already want to use another line besides AB, there is no need for CE itself.

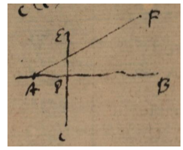

Figure 12

And so we can abstain from motion and conceive that the line CD intersects the line AB perpendicularly, and taking some number of points on the line AB and in CD equidistant from E, join these points. All such lines will give a plane, and because CD relates to the plane in the same way on both sides, this plane will be intersected by the line in such a way that it relates in the same way on both sides, and so any line intersects a plane in the same way on both sides.

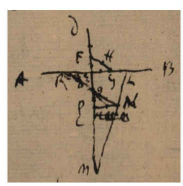

Figure 13

Hence a plane is the locus of points relating uniquely to three points not on lying a line ACE. Indeed, F relates uniquely to CE, and G relates uniquely to CE, and G relates uniquely to $A.E$, therefore $F.G$ relates uniquely to A, C, E; now H relates uniquely to F,G. Therefore it also relates uniquely to A, C, E. [[1]From this construction the plane is also uniform everywhere with respect to the line AB.]

But one would need the converse, and one must show that the plane is the locus of all such points or that a line, two points of which fall in

[1]Leibniz cancelled this sentence.

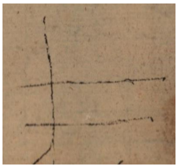

Figure 14

a plane, also falls in the plane itself. Taking any points L, M in the lines AB, CD, one must show that a point of line LM, say N, falls in the plane. Let NP be drawn perpendicularly from L into CE and take $PZ = PN$; NZ being joined, it will meet BE in R. And the line RZ is one of the previous ones, which make up the plane. But one needed to show that RZ and NZ fall in [the same] direction.