G. W. Leibniz

Specimen Analyseos Figuratae

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Specimen Analyseos Figuratae in *Elementis* Geometriae

Analysin figuratam voco, quae modum praestat literis puncta significantibus repraesentandi Figuras, et inveniendi atque demonstrandi earum effectus et proprietates, ita ut non tantum magnitudines, ut in Calculo Algebraico, sed situs ipsi per novum hoc Calculi genus, directe exhibeantur. Specimen autem hujus artificii edemus in *Elementis* Euclidis, et inter procedendum assumemus Lemmata, Axiomata, Definitiones aliasque propositiones, quibus indigebimus.

Quicquid autem in Decem prioribus libris Euclidis exponitur, hoc intelligendum est *in uno esse plano*. Ne id perpetuo admoneri opus sit.

Ad Lib. 1 Elem.

Prop. 1. Super data recta linea terminata AB. triangulum aequilaterum ABC construere.

(1) $AC = AB$ ex hypothesi. (2) $BC = BA$ ex hyp. Et quod his

Specimen Analyseos Figuratae in *Elementis* Geometriae

I call figurate analysis that which affords a method of representing figures by letters signifying points, and of discovering and demonstrating their effects and properties, so that not only magnitudes, as in the algebraic calculus, but also situations themselves are directly exhibited by this new type of calculus. Here we will give a sample of this technique in the *Elements* of Euclid, and in the process we will take up the lemmas, axioms, definitions, and other propositions we need.

Now whatever is explained in the first 10 books of Euclid, this is to be understood *to be in one plane* — so we won't need to be constantly reminded of this.[1]

For Book I of the Elements

Proposition 1. Over a given straight line with endpoints AB, to construct the equilateral triangle ABC.

(1) $AC = AB$ by hypothesis. (2) $BC = BA$ by hypothesis. And C is that

[1]Leibniz believes that space, where extended things have place, is essentially 3-dimensional—but one can do geometry in a plane within space.

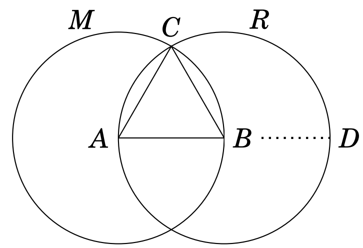

Figure 1

satisfacit est C. (3) Sit $AM = AB$. (4) Fiat (per postulat. 1) $\overline{M}$ *circumferentia circuli centro A intervallo AB* (per definitionem 1). (5) Sit $BR = BA$. (6) Fiat $\overline{R}$ circumf. circuli centro B intervallo BA (ut ad 4). (7) Jam quoddam R est M (per Lemma 1) (8) seu $\overline{R}$ et $\overline{M}$ *sibi occurrunt* (ex. 7. per defin. 2). (9) Datis lineis $\overline{M}$ et $\overline{R}$ (per 4 et 6) sibi occurrentibus (per 7, 8) habetur eorum occursus (per *postulatum* 2) nempe R quod est M. (10) Id vero est C (per 1 et 3 ac per 2 et 5). (11) Habetur ergo C. (12) Jam datur AB (ex hyp.). (13) Ergo habetur ABC. Qu. Er. Fac.

which satisfies these. (3) Let $AM = AB$. (4) Then (by Postulate 1[2]) $\overline{M}$ would be *the circumference of a circle with center A and interval AB* (by Definition 1). (5) Let $BR = BA$. (6) Then $\overline{R}$ would be the circumference of a circle with center B and radius BA (as in 4). (7) Now some R is M (by Lemma 1). (8) That is, $\overline{R}$ and $\overline{M}$ meet each other (from 7 by Definition 2). (9) Given curves $\overline{R}$ and $\overline{M}$ (by 4 and 6) meeting each other (by 7, 8), one has their meeting (by Postulate 2), namely an R which is an M. (10) But this is C (by 1 and 3 and by 2 and 5). (11) Therefore we have C. (12) AB is already given (by hypothesis). (13) Therefore we have ABC. Q.E.F.

[2]Leibniz collected two lists of postulates, definitions, and axioms in the margins of the first two manuscript pages. We have provided the two lists in boxed portions.

Def. 1. Circumf. centr. interv. lib. 1. pr. 1 art. 4.	*Definition 1:* Circumference, center, interval. Book 1, Proposition 1, item 4.
Postul. 1. Dato centro et intervallo circulum describere.	*Postulate 1:* To describe a circle with given center and interval.
Def. 2. Occursus art. 8.	*Definition 2:* The meeting. Item 8.
Postul. 2. Datis occurrentibus habetur occursus art. 9.	*Postulate 2:* Given things that meet, one has the meeting. Item 9.
Def. 3. Intra circulum esse, art. 16.	*Definition 3:* Being inside a circle. Item 16.
Postul. 3. Recta produci potest ex uno termino ad distantiam quantamvis art. 17. 26.	*Postulate 3:* A line can be extended from one endpoint to any distance whatever. Items 17, 26.
Axiom 1. Totum majus parte art. 20.	*Axiom 1:* The whole is greater than the part. Item 20.
Def. 4. Extra circ. esse art. 21.	*Definition 4:* Being outside a circle. Item 21.
Ax. 2. Continuum quod intra et extra figuram est ejus circumferentiae occurrit art. 24.	*Axiom 2:* A continuum that is inside and outside a figure meets its circumference. Item 24.
Def. 5. Recta est via brevissima inter extrema sive distantia duorum punctorum. Art. 32.	*Definition 5:* A straight line is the shortest path between extremes, or the distance of two points. Item 32.
Ax. 3. Omnes partes nullam partem communem habentes aequantur Toti. Art. 34.	*Axiom 3:* All parts, having no common part, equal the whole. Item 34.
Ax. 4. Quod movetur non est simul in pluribus locis. Art. 38.	*Axiom 4:* What moves is not in multiple places simultaneously. Item 38.

Lemma 1. Si duo circuli MA ex A et RA ex B habeant mutuo centrum in alterius circumferentia B in $\overline{M}$ et A in $\overline{R}$ eorum circumferentiae $\overline{M}$ et $\overline{R}$ sibi occurrent alicubi in C.

Lemma 1. If two circles MB[3] from A and RA from B each have their center in the other's circumference, B in $\overline{M}$ and A in $\overline{R}$, their circumferences meet someplace, at C. [See Figure 1.]

Iisdem quae antea positis, (14) $BA = BA$ (per se). (15) Ergo quoddam R est A (per 5). (16) Itaque quodd. R est intra circulum $A.\overline{M}$ (per defin. 3, nam *punctum intra circulum esse* dicimus cujus distantia a centro est minor radio). (17) Producatur AB ex B in D (per postulatum 3) (18) ut sit $BD = BA$ (per Lemma 2). (19) Ergo $AD = AB + BD$ (per Lemma 3). (20) Ergo $AD \sqcap AB$ (totum parte per Axiom. 1). (21) Ergo D est extra $A.\overline{M}$ (per def. 4, nam *punctum extra circulum esse* dicimus cujus distantia a centro est major radio). (22) Jam D est R (per 18 et 5). (23) Ergo qu. R est extra $A.\overline{M}$. (24) Itaque (per 16 et 23) qu. R est in $\overline{M}$ (per Ax. 2, omne continuum $\overline{R}$ quod intra et extra figuram $A.\overline{M}$ est, esse et in

Supposing the same things as before, (14) $BA = BA$ (per se), (15) therefore some R is A (by 5). (16) And so some R is inside the circle $A.\overline{M}$ (by Definition 3, since we say *a point is inside a circle* if its distance to the center is less than the radius). (17) Let AB be extended from B to D (by Postulate 3) (18) so that $BD = BA$ (by Lemma 2). (19) Therefore $AD = AB + BD$ (by Lemma 3). (20) Therefore $AD \sqcap AB$ (the whole [is greater] than the part, by Axiom 1). (21) Therefore D is outside $A.\overline{M}$ (by Definition 4, indeed we say *a point is outside a circle* if its distance from the center is greater than the radius). (22) But D is R (by 18 and 5). (23) Therefore, some R is outside $A.\overline{M}$. (24) And so (by 16 and 23) some R is on $\overline{M}$ (by Axiom 2, every continuum $\overline{R}$ which is both inside and outside the figure $A.\overline{M}$, is also on its circumference

[3] Leibniz wrote MA. It is possible, if less likely, that he intended "MA from A and RB from B."

ejus circumferentia $\overline{M}$. *Schol*. Unde etsi vi formae ex puris particularibus nil sequatur, tamen vi materiae in continuis ex 16 et 23 sequitur 24.)

(25) Ergo (ex 24 per def. 2. ad 8) $\overline{M}$ et $\overline{R}$ sibi occurrunt. Q. E. D.

Lemma 2. Recta AB ex centro B (imo puncto quovis intra Circulum) produci potest ut et circumferentiae circuli $\overline{R}$ alicubi occurrat in D.

(26) Produci enim potest ad distantiam quantamvis (per postul. 3. ad 17). (27) Ergo ad E sic ut sit $BE \sqcap BA$. (28) Ergo (per def. 4. ad 21) recta producta est extra circulum $B.\overline{R}$. (29) Eadem est in circulo ad ejus centrum B (per def. 3. ad 16). (30) Ergo (per Ax. 2. ad 23) occurrit ejus circumferentiae alicubi in D.

Coroll. Lemmatis 2. Recta AD transiens per punctum B intra circulum, circulo bis occurrit in A et D. Nam AB producta ex B (recedendo ab A) circumferentiae occurrit alicubi in D, per Lemm. 2. Et DB producta ex B (recedendo a D) circulo occurret alicubi in A.

Lemma 3. Si tria puncta $A.B.D.$ sint in recta distantia duorum quorundam ex ipsis, AB coincidit ipsi $AB + BD$ summae distantiarum tertiae D ab ipsis A, B.

$\overline{M}$. *Scholium:*[4] Whence,[5] although nothing would follow from the pure particulars in virtue of form, yet in virtue of material in continua, 24 follows from 16 and 23.)

(25) Therefore, (from 24 by Definition 2 at 8) $\overline{M}$ and $\overline{R}$ meet each other. Q.E.D.

Lemma 2. Line AB from center B (indeed from any point inside the circle) can be extended so that it also meets the circumference of the circle $\overline{R}$ someplace, at D.

(26) For it can be extended to an arbitrarily large distance (by Postulate 3 at 17). (27) Therefore [it can be extended] to E such that $BE \sqcap BA$. (28) Therefore, (by Definition 4 at 21) the line has been extended outside the circle $B.\overline{R}$. (29) The same line is in the circle at its center (by Definition 3 at 16). (30) Therefore (by Axiom 2 at 24[6]), it meets its circumference someplace, at D.

Corollary of Lemma 2.[7] A line AD passing through a point B inside a circle meets the circle twice, at A and D. For the line AB extended from B (receding from A) meets the circumference someplace, at D, by Lemma 2. And DB extended from B (receding from D) will meet the circle someplace, at A.

Lemma 3.[8] If three points A, B, D[9] are on a line, the distance of some two of them, say AD, coincides with the sum $AB + BD$ of the distances of the third[10] B from A, D.

[4] A *scholium* is a sort of interpretive marginal note or explanatory comment.
[5] An illegible word follows here and may have been crossed out.
[6] Manuscript has "at 23".
[7] In the manuscript, this is written in the margin beside Lemma 3.
[8] Leibniz wrote two versions of this lemma and proof. In the first version, A and B were the endpoints, while in the second version, A and D were the endpoints. (In the first version he still wrote $AB + BD$, which was inconsistent with his setup.) Then Leibniz crossed out the proof in the first version and the statement in the second version. We have opted to switch B and D where appropriate to make the statement and proof consistent.
[9] The manuscript has "$A.B.D$", which appears to be a reversion to a notation with periods that Leibniz uses elsewhere but has otherwise avoided in this essay.
[10] In the Latin, the word "third" grammatically matches "sum" rather than "D"; we believe this may be a typo, but in any case the meaning of the overall statement is clear.

(31) Tria puncta sunt in recta (ex Hyp.). (32) *Recta AD* est via brevissima seu distantia inter extrema A et D (per def. 5). (33) Ergo punctum B in recta minus distat ab extremo A, quam extrema A, D inter se. (34) Et $AB + BD = AD$ (omnes partes nullam partem communem habentes toti, per ax. 3). (35) Est autem AB distantia inter A et B et BD inter B et D (per def. 5. ad 32). (36) Superest ut ostendamus tribus punctis in recta existentibus capi posse rectam quae duobus ex illis terminetur tertium vero includat. (37) Ponamus enim punctum aliquod rectam cui insunt percurrere. (38) Id ipsa attinget successive (per axioma 4). (39) Esto ergo A primum, B secundum, D tertium. (40) Ergo portio rectae quam percurret inter A et D terminabitur ipsis A et D, comprehendet autem B.

(31) The three points are on a line (by hypothesis). (32) The *line AD* is the shortest path or the distance between the extremes A and D (by Definition 5). (33) Therefore the point B on the line is less distant from the extreme A than the extremes A and D are from each other. (34) And $AB + BD = AD$[11] (all the parts having no common part [are equal] to the whole by Axiom 3). (35) But AB is the distance between A and B, and BD the distance between B and D (by Definition 5 at 32). (36) It remains for us to show that when three points exist on a line, one can get a line which ends in two of them while including the third. (37) Indeed let us suppose that some point runs along the line which they are in.[12] (38) It will reach them successively (by Axiom 4). (39) Therefore let A be the first, B the second, D the third. (40) Therefore the portion of the line it traverses between A and D will terminate in A and D but include B.

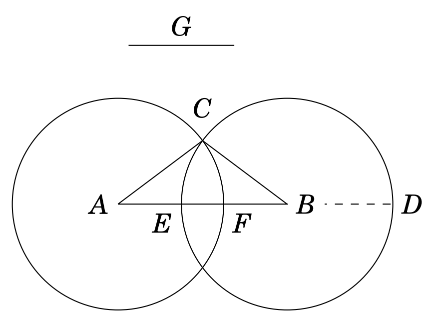

Figure 2

Additamentum 1. Si duo circuli $A\overline{M}$, $B\overline{R}$ aequalium radiorum habeant radium, majorem dimidia distantia centrorum A. B. sibi occurrent in C extra rectam per centra.

Addition 1. If two circles $A\overline{M}$, $B\overline{R}$ of equal radii have radius greater than half the distance of their centers $A.B$[13], they will meet each other in C outside the line through the centers.

[11] This equality, for Leibniz, means that the sum of the lengths of AB and BD is the same as the length of AD. A sum can be taken even when the summands are not parts of a common whole. It seems that what Leibniz thinks is still needed at this point is to show that the line AD (the shortest path) and the line on which A, B, D are given must be one and the same line.

[12] In certain writings, arguably especially in the range 1685-95, Leibniz uses "inesse" (appearing here) with a technical meaning as part of his mereological calculus.

[13] The "$A.B$" was inserted later, and this fact may explain the dot.

(41) Recta AB secat $\overline{R}$ bis in E et D (per corroll. Lemm. 2). (42) Et similiter $\overline{M}$ in F. (43) Sit E ex B versus A (44) et D ex B recedendo ab A (45) sitque F ex A versus B (46) AE erit minor AF (adde mox 58). (47) Nam quia (ob 43) E cadit inter B et A, vel B inter E et A (48) erit (per Lemm. 3) AE differentia inter AB et BE, (49) seu inter AB et AF ((50) quia $AF = BE$ (ex Hyp.)). (51) Jam si $AF \ulcorner AB$ (52) erit $AF = AB + AE$ (per 48, 49). (53) Ergo $AF \ulcorner AE$. (54) Sin $AB \ulcorner AF$, erit $AB - AF = AE$ (per 48, 49). (55) Jam $AF \ulcorner \frac{1}{2}AB$ (ex Hyp.). (56) Ergo $AE \urcorner \frac{1}{2}AB$. (57) Ergo $AF \ulcorner AE$. (58) Utroque ergo modo erit $AE \urcorner AF$ ut asserebatur art. 46. (59) Rursus AD est major AF. (60) Nam $AD = AB + BD$ (per Lemm. 3) (61) $= AB + AF$ (quia $BD = AF$ ex hyp.). (62) Ergo quoddam R est intra $\overline{M}$ nempe E. (quia $AE \urcorner AF$ seu AM per 58). (63) Quoddam R est extra $\overline{M}$ nempe D (quia per 59 $AD \ulcorner AM$ seu AF). (64) Ergo (per Ax. 2) Qu. R est in $\overline{M}$, ut C.

Additamentum 2. Super basi data AB. triangulum isosceles construere cujus crura AC vel BC sint magnitudinis datae G, quam oportet esse majorem dimidia basi AB.

(65) Centris A et B (66) intervallis $= G$ (per prop. 2. independenter ab hac demonstratam) (67) describantur circuli (postul. 1.) se secantes alicubi in C. Per additam. 1. erit $AC = G$ et $BC = G$. Quod erat fac.

Prop. 2. Ad datum punctum A ponere datae rectae BC aequalem rectam AG.

Solutio. (1) Jungatur AC. (2) Super qua Triang. Aequil. ACD

(41) The line AB intersects $\overline{R}$ twice, at E and D (by the corollary of Lemma 2). [See Figure 2.] (42) And similarly $\overline{M}$ at F. (43) Let E be [on the line] from B toward A (44) and let D be [on the line] from B moving away from A (45) and let F be [on the line] from A toward B, (46) AE will be less than AF (see 58 shortly). (47) Since (in view of 43) E falls between B and A, or B between E and A, (48) AE will be (by Lemma 3) the difference between AB and BE, (49) or between AB and AF, (50) since $AF = BE$ (by the hypothesis). (51) Now if $AF \ulcorner AB$ (52) we will have $AF = AB + AE$ (by 48, 49). (53) Therefore, $AF \ulcorner AE$. (54) But if $AB \ulcorner AF$, we will have $AB - AF = AE$ (by 48, 49). (55) Now $AF \ulcorner \frac{1}{2}AB$ (by the hypothesis). (56) Therefore $AE \urcorner \frac{1}{2}AB$. (57) Therefore $AF \ulcorner AE$. (58) Either way, therefore, $AE \urcorner AF$ as was asserted in item 46. (59) In turn, AD is greater than AF. (60) For $AD = AB + BD$ (by Lemma 3) (61) $= AB + AF$ (because $BD = AF$[14] by hypothesis). (62) Therefore some R is inside $\overline{M}$, namely E (because $AE \urcorner AF$ or AM by 58). (63) Some R is outside $\overline{M}$, namely D (since by 59, $AD \ulcorner AM$ or AF[15]). (64) Therefore (by Axiom 2) some R is on $\overline{M}$, say C.

Addition 2. Above a given base AB, construct an isosceles triangle whose legs AC or BC are of a given magnitude G, which should be greater than half the base AB. [See Figure 2.]

(65) With centers A and B, (66) interval $= G$ (by Proposition 2 demonstrated independently of this), (67) let circles be described (Postulate 1) which intersect someplace, at C. By Addition 1 we will have $AC = G$ and $BC = G$. Q.E.F.

Proposition 2. At a given point A place a line AG equal to a given line BC. [See Figure 3.]

Solution. (1) Connect AC. (2) Let equilateral triangle ACD be con-

[14] The manuscript has "$BD = BF$" but the argument suggests it should be $BD = AF$.

[15] "AM" in the manuscript.

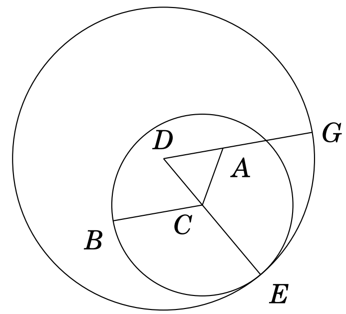

Figure 3

construatur (per 1. prim.). (3) Centro C. radio CB describatur circumferentia circularis BE (per postul. 1.) (4) cui recta DC producta ex C (per postul. 3) (5) occurret alicubi in E (per prop. 1. Lemm. 2). (6) Centro D radio DE describatur Circulus (postul. 1) (7) cui recta DA producta ex A occurret alicubi in G (per dict. Lemm. 2). (8) Erit $AG = BC$. (9) Nam $DC + CE = DE$ (per 5 et Lemm. 3 ad prop. 1.) (10) $= DG$ (per 6 et 7) (11) $= DA + AG$ (per 7 et Lemm. 3 ad prop. 1.) (12) $= DC + AG$ (per 2). (13) Ergo (per 9 et 12) $AG = CE$ (14) $= BC$ (per 3. 4).

Schol. ad prop. 2. Analysis qua constructio haec inveniri potest, talis est. Ad punctum A ponenda est recta aequalis rectae positae ad punctum C. Recta autem posita ad punctum C intelligi potest non tantum BC, sed et alia quaecunque ut CE, ipsi CB aequalis, seu ex C ad circumferentiam circuli CBE ducta. Cum ergo puncta A et C debeant tractari eodem modo, etiam recta AC tractanda est sic, ut respectu C tractetur quemadmodum tractata est respectu A. Quaeratur ergo punctum aliquod D eodem modo se habens ad A et C, quod fit

structed upon it (by [Proposition] 1 of the first [Book][16]). (3) Let a circular circumference BE be constructed with center C, radius BC (by Postulate 1), (4) which the line DC extended from C (by Postulate 3) (5) will meet someplace, at E (by Proposition 1, Lemma 2). (6) Let a circle be described with center D, radius DE (Postulate 1), (7) which the line DA extended from A will meet someplace, at G (by the assertion in Lemma 2). (8) We will have $AG = BC$. (9) Indeed $DC + CE = DE$ (by 5 and Lemma 3 for Proposition 1). (10) $= DG$ (by 6 and 7) (11) $= DA + AG$ (by 7 and Lemma 3 for Proposition 1). (12) $= DC + AG$ (by 2). (13) Therefore (by 9 and 12) $AG = CE$ (14) $= BC$ (by 3, 4).

Scholium to Proposition 2.[17] The analysis by which this construction can be discovered is like so: A line is to be placed at point A equal to the line placed at point C. By a line placed at point C one can understand not only BC but also any other such as CE, equal to CB, or drawn from C to the circumference of a circle CBE. Since, therefore, points A and C ought to be treated in the same way, the line AC should also be treated in such a way that with respect to C it is treated just as it has been treated with respect to A. Therefore, let some point D be sought relating in the same way to A and C, which arises by constructing an

[16]The manuscript has elliptical references to Euclid; here "1. prim".

[17]This scholium is revealing about Leibniz's interests. He considers local procedure in an abstract light, focusing on symmetry in the mode of treatment and argument about parts of the construction. A symmetry in what is given should engender a symmetry in what is determined according to the same mode of treatment. See also the argument in Prop. 8 with his consideration there of things "treated in the same way" and "relating in the same way".

super AC construendo triangulum aequilaterum vel isosceles ADC. Ducta recta ex D per C producta occurret circulo in E; ipsi DE sumatur aequalis DG in recta ex D producta per A, erit et $AG = CE$, quia eodem modo invenitur G respectu A, quo E respectu C.

equilateral or isosceles triangle ADC. The line drawn from D through C will meet the circle in E; in the line from D extended through A let DG be taken equal to DE, and we will have $AG = CE$, because G is found in the same way with respect to A as E with respect to C.

Prop. 3. Duabus datis rectis A et BC majore ab ea detrahere BE aequalem ipsi A.

Proposition 3. Given two lines, A and a larger BC, from [BC] take off BE equal to A. [See Figure 4.]

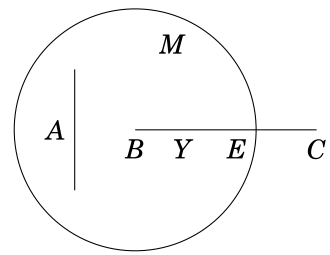

Figure 4

(1) Fac $BD = A$ (per 2. prim.) (2) et fac $\overline{M}$ ut sit $BM = BD$ (per postul. 1.). (3) Jam BC est intra $\overline{M}$ in B (ut ad art. 16. prop. 1.) (4) extra $\overline{M}$ in C ((5) quia $BC \sqcap A$ ex hyp. (6) ergo $\sqcap BM$ per 1. 2.) (6) Ergo (per Ax. 2) BC occurrit ipsi $\overline{M}$ alicubi in E. (7) Ergo BE pars BC. (8) Et $BE = BD = A$.

(1) Make $BD = A$ (by [Proposition] 2[18] of the first [Book]) (2) and make $\overline{M}$ such that $BM = BD$ (by Postulate 1). (3) Now BC is inside $\overline{M}$ at B (as at item 16 of Proposition 1) (4) and outside $\overline{M}$ at C ((5) because $BC \sqcap A$ by hypothesis, (6) hence $\sqcap BM$ by 1, 2). (6) Therefore (by Axiom 2) BC meets $\overline{M}$ someplace, at E. (7) Therefore BE is a part of BC. (8) And $BE = BD = A$.

Def. 6. Si A sit aequale parti ipsius B, A *minus* dicetur B *majus*. Lib. 1. pr. 3. art. 6.	*Definition 6:* If A is equal to a part of B, then A is said to be *lesser*, B *greater*. Book 1, Proposition 3, item 6.
Def. 7. Anguli rectilinei aequales dicuntur si congrui sunt. Prop. 4. Art. 3.	*Definition 7: Rectilinear angles* are said to be *equal* if they are congruent. Proposition 4, item 3.
Ax. 5. Datis punctis ad quae anguli figurae consistunt figura data est. Art. 9. Potest censeri postulatum.	*Axiom 5:* Given the points at which the angles of a figure stand, the figure is given. [Proposition 4,] item 9. This can be considered a postulate.
Ax. 6. Quae eodem modo ex datis congruis (determinate) dantur congrua sunt. Art. 11.	*Axiom 6:* Those which are given (determinately) in the same way from congruent givens are congruent. [Proposition 4,] item 11.

[18] Leibniz's reference "3. prim." seems to indicate the wrong proposition.

Item sic:

(3) Sit $\overline{Y} \infty BC$. (4) Erit $BY + YC = BC$ (per Lemm. 3. ad prop. 1.). (5) $BC \sqcap A$ (ex hyp.). (6) Ergo qu. $BY = A$ (nam ex *definitione* 6. Majoris et Minoris, *Minus* est A cui qu. BY pars alterius, BC, quod *majus* dicitur, aequalis est). (7) Ergo qu. Y est M. (9) Quod sit E. Datur ergo E (per postul. 2.). (10) Ergo et $BE = A$ (per 6) (11) pars BC (per 4). Q. E. F.

Also thus:

(3) Let $\overline{Y} \infty BC$. (4) We will have $BY + YC = BC$ (by Lemma 3 for Proposition 1). (5) $BC \sqcap A$ (by hypothesis). (6) Therefore some $BY = A$ (indeed by *Definition 6*. Greater and lesser, A is *less* when some part BY of the other, BC, which is called *greater*, is equal to it). (7) Therefore some Y is M. (9) Let it be E. Therefore some E is given (by Postulate 2). (10) Therefore also $BE = A$ (by 6), (11) a part of BC (by 4). Q.E.F.

Prop. 4. Si duo Triangula BAC, EDF, duo latera unius BA, AC duobus lateribus alterius ED, DF, respondens respondenti, aequalia habeant, et angulum unius A angulo alterius D aequalem, sub aequalibus rectis lineis contentum; tunc Triangulum triangulo congruum erit.

Proposition 4. If two Triangles BAC, EDF have two sides of the one, BA, AC equal to two corresponding sides of the other, ED, DF respectively, and the angle A of the one equal to the angle D of the other, contained by the equal straight lines, then the one triangle will be congruent to the other. [See Figure 5.]

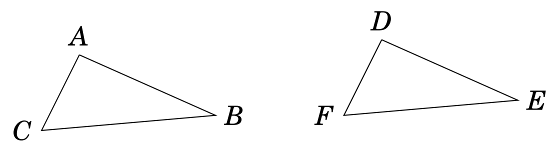

Figure 5

(1) Ponamus dari positione Angulos LAM et NDP, (2) aequales ex Hypoth. (3) ergo congruos. (Nam *anguli rectilinei aequales* definiuntur *def*. 7. qui congrui sunt.) (4) Et G magnitudinem AC et DF, (5) et H magnitudinem AB et DE. (6) Denique dari in quibus lateribus angulorum sumendae sint hae rectae, nempe AB in AM, AC in AL, DE in DP, DF in DN. (7) Dantur ergo puncta B, C, item E, F (per 3. primi). (8) Ergo A, B, C; item D, E, F (per 1 et 7.). (9) Ergo Triangula ABC et DEF (nam datis punctis ad quae anguli figurae consistunt, figura data est per Ax. 5.). (10) Et quidem ambo eodem modo ex datis congruis determinate exhibentur (ex toto processu).

(1) Let us suppose that the angles are given in position, LAM and NDP, (2) equal by the hypothesis, (3) hence congruent. (For *equal rectilinear angles* are defined in *Definition 7* as those which are congruent.) (4) And [suppose] G [has] the magnitude of AC and DF (5) and H the magnitude of AB and DE. (6) Finally, it is given on which sides of the angles these lines are to be taken, namely AB in AM, AC in AL, DE in DP, DF in DN. (7) Therefore points B, C, likewise E, F are given (by [Proposition] 3 of the first [Book]). (8) Therefore A, B, C [are given]; likewise D, E, F (by 1 and 7). (9) Therefore, triangles ABC and DEF [are given] (for, given the points at which the angles of the figure stand, the figure is given by *Axiom 5*). (10) And indeed both are exhibited

(11) Ergo congrua sunt (per *Axioma* 6). Q. E. D.

determinately in the same way from congruent givens (by the whole procedure). (11) Therefore they are congruent (by *Axiom 6*). Q.E.D.

Schol. Si quis superpositionem adhibeat res eodem redit, si enim data congrua fiant actu congruentia seu coincidentia per superpositionem, coincident etiam quae ex ipsis determinate dantur; alioqui non unum sed plura his datis satisfacientia haberi possent, contra hypothesin. Unde Axiomatis sexti ratio intelligitur.

Scholium. If one applies superposition, it comes to the same thing; indeed, if the congruent givens become actually congruous, or coincident, by superposition, then those which are given determinately[19] from them will also coincide; otherwise not one but several things satisfying these givens could be found, contrary to hypothesis. From this one understands the reason for the sixth axiom.

Porisma. Triangulum magnitudine et specie datur magnitudine datis angulo et lateribus eum comprehendentibus (per 1. usque ad 9). Nam ut positione detur tantum opus est angulum positione dari, et quae crura anguli, quibus magnitudinibus laterum assignentur, quae nihil in magnitudine et specie mutant, nam utrumque crus in angulo eodem modo se habet.

Porism. A triangle is given in magnitude and shape[20] if an angle and the sides enclosing it are given in magnitude (by 1 through 9). Indeed, for it to be given in position, one only needs an angle to be given in position, as well as which legs of the angle are assigned to which magnitudes of sides; this changes nothing in the magnitude and shape, since either leg relates in the same way at the angle.

Schol. *Porisma* voco quod ex demonstratione colligitur, *corollarium* quod ex propositione.

Scholium. I call *porism* that which is inferred from a demonstration, and *corollary* that inferred from a proposition.

Prop. 5. Triangulum ABC, quod duo latera AB, AC aequalia habet, etiam angulos eorum B, C ad reliquum latus, BC, aequales habet.

Proposition 5. A triangle ABC that has two sides AB and AC equal also has their two angles B and C on the remaining side BC equal. [See Figure 6.]

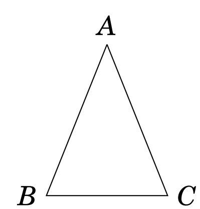

Figure 6

[19]For Leibniz, something given by a geometric procedure or construction is given *determinately* when it is unique among things satisfying the conditions, *semideterminately* when it is one of a finite (or discrete) set of things, and *indeterminately* when it is one of an infinite (or continuous) set of things satisfying the conditions.

[20]Leibniz's word *species* can mean both "appearance" in the sense of form or shape, and "species" by contrast with genus or family.

(1) Ponatur positione data *BC*. (2) Et data sit *G*, aequalis ipsis *BA*, *CA* (3) et *partes* ad quas cadere debet *A* (hoc est per *def*. 8. plano secto in duas partes a recta *BC* indefinite producta detur in qua parte debeat esse *A*). (4) Datur *A* si est possibilis, (5) nam centris *B* et *C* intervallo aequali *G* (per 2. primi) (6) describuntur circuli, (postul. 1) (7) jam *A* est in circumferentia utriusque (per 2) si scilicet possibilis est. (8) Ergo circuli sibi occurrunt in *A* si *A* est possibilis. (9) Ergo (per postul. 2.) datur *A*. (10) Ergo (per ax. 5) datur *ABC*. (10) Ergo et anguli *ABC*, *ACB* (nam per ax. 7 dato aliquo dantur ejus requisita) (11) et quidem eodem modo (per processum expositum). (12) Ergo (per Ax. 6) congrui sunt anguli. (13) Ac proinde aequales. (Nam congrua sunt aequalia per ax. 8.)

(1) Suppose *BC* is given in position. (2) And let *G* be given, equal to *BA*, *CA* (3) and the *side* on which[21] *A* should fall (this is by *Definition 8*: the plane being cut into two parts by the line *BC* extended indefinitely, let it be given in which part *A* should be[22]). (4) *A* is given if it is possible, (5) since, with centers *B* and *C* and interval equal to *G* (by [Proposition] 2 of the first [Book]), (6) circles are described (Postulate 1). (7) Now *A* is in the circumference of both (by 2), if it is possible, of course. (8) Therefore the circles meet each other at *A* if *A* is possible. (9) Therefore (by Postulate 2) *A* is given. (10) Therefore (by Axiom 5) *ABC* is given. (10) Therefore also the angles *ABC*, *ACB* (since by Axiom 7, when something is given, its requisites are given) (11) and indeed, in the same manner (by the procedure explained). (12) Therefore (by Axiom 6) the angles are congruent. (13) And hence equal. (For congruents are equals by Axiom 8.)

Schol. Idem ostendi potuisset per superpositionem, si aliud sumtum fuisset triangulum huic congruum, *DEF* et nunc *ABC* applicaretur ipsi *DEF*; nunc *ACB* ipsi *DEF*, ita nunc angulus *ABC* nunc angulus *ACB* congrueret eidem *DEF*, ergo congrui essent inter se.

Scholium. The same thing could have been shown by superposition, if some triangle *DEF* had been taken congruent to this one, and now *ABC* would be placed onto *DEF*, now *ACB* onto *DEF*; thus now angle *ABC*, now angle *ACB*, would agree with the same *DEF*; therefore, they would be congruent to each other.

Prop. 6. Triangulum *ABC* quod duos angulos *B* et *C* aequales habet, etiam latera *AB*, *AC* ad reliquum angulum *A* pertinentia aequalia habebit.

Proposition 6. A triangle *ABC* that has two angles *B* and *C* equal will also have the two sides *AB* and *AC* belonging to the remaining angle *A* equal.

Demonstratur eodem modo, (1) quia dato latere *BC*, et angulis *B*, *C* et partibus ad quas debet esse *A*, datur *A*. (2) Nam data positione recta *BC* et angulo ad eam alterius rectae *BA*[,] datur ipsa recta *BA*, indefinite (dato enim positione angulo latera dantur per ax. 7), eodem modo *CA* indefinite, (3) quae si possibilis est *A* se secabunt in *A*. (4) Datur ergo *A*, adeoque utraque eodem modo, ergo congruent. Q. E. D.

This is demonstrated in the same way, (1) since, given side *BC* and angles *B*, *C* and the side on which *A* should be, *A* is given. (2) Indeed, line *BC* and the angle to it of another line *BA* being given in position, the line *BA* itself is given indefinitely (for the angle being given in position, by Axiom 7 the sides are given), in the same way *CA* indefinitely; (3) these will intersect each other in *A* if *A* is possible. (4) Therefore *A* is given, and thus both [sides are given] in the same way, therefore

[21]Latin “partes ad quas”. Similarly in item 1 of Prop. 6 below.
[22]This definition did not make it into the marginal lists.

they will be congruent. Q.E.D.

Schol. Idem potuisset demonstrari ex praecedenti. Ut et per superpositionem ad instar praecedentis.

Scholium. The same thing could have been demonstrated from the preceding. As also by superposition in the manner of the preceding.

Prop. 7. Si triangula ABC, DEF latera lateribus alterius aequalia habeant, respondens respondenti, Triangula erunt congrua.

Proposition 7. If triangles ABC, DEF have sides equal to the corresponding sides of the other, the triangles will be congruent. [See Figure 7.]

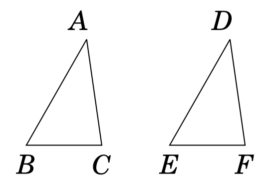

Figure 7

Demonstratur eadem Methodo, quia data positione uno latere unius trianguli, et uno alterius aequali, et reliquis datis magnitudine, circulos ex datorum positione laterum extremis, et magnitudinibus tanquam intervallis describendo dabitur utrumque triangulum, eodem modo, ergo per Ax. 6. congrua erunt.

This is demonstrated by the same method, since, one side of one triangle being given in position, and the one equal to the other, and the remaining ones being given in magnitude, by describing circles from the extremes of the sides given in position and with the magnitudes as intervals, each of the triangles will be given, in the same way; therefore by Axiom 6 they will be congruent.

Prop. 8. Datum Angulum BAC bisecare.

Proposition 8. To bisect a given angle BAC. [See Figure 8.[23]]

Sit recta bisecans FA, et $FAB = FAC$. Ea se eodem modo habet ad BA et ad CA. Habemus unum punctum rectae FA, nempe A, quaeratur adhuc aliud F ad quod ea recta se habeat ut ad A. Quod fiet si BAC, (positis BA, CA, aequalibus ut utrumque latus eodem modo tractetur respectu FA) transferamus in BFC centris B et C, radiis aequalibus ipsi BA vel CA describendo circulos, qui se secabunt alicubi in F (quia

Let FA be the bisecting line, and $FAB = FAC$. It relates in the same way to BA and CA. We have one point of the line FA, namely A, let us further seek some F to which this line relates as to A. This will happen if (supposing BA, CA are equal so that each side is treated in the same way with respect to FA) we translate BAC into BFC by describing circles with centers B and C and radii equal to BA or CA[24], which intersect each other someplace, at F (since triangle BFC is congruent

[23]Leibniz originally had the lines from A to F, B to F, and C to F dashed, but then he drew over them with a solid line.
[24]Leibniz wrote CB, presumably by mistake.

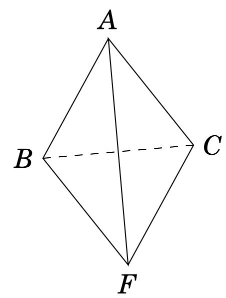

Figure 8

triangulum BFC ipsi BAC congruum per 7. prim. ob basin eandem BC et latera aequalia, utique possibile est). Recta ergo FA eodem modo se habens ad AB et AC utique angulum bisecabit.

to BAC by [Proposition] 7 of the first [Book] on account of the same base BC and equal sides, certainly it is possible). Therefore line FA, relating in the same way to AB and AC, will certainly bisect the angle.

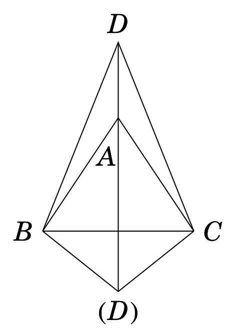

Figure 9

Idem praestabitur si super BC basi trianguli isoscelis ABC quodcunque aliud triangulum isosceles BFC construamus per additament. 2. ad prop. 1. Nam locus omnium punctorum eodem modo se habentium ad duo latera ejusdem anguli, vel ad duo extrema ejusdem rectae, est recta transiens per duos apices duorum isoscelium eodem modo se ad angulum vel

The same will obtain if we construct any isosceles triangle BFC whatsoever over the base BC of the isosceles triangle ABC, by Addition 2 to Proposition 1. For the locus of all points relating in the same way to the two sides of the same angle or to the two extremes of the same line is a line passing through the two apices of the two isosceles triangles relating in the same way to the proposed angle or line.

rectam propositam habentium.

Prop. 9. Datam rectam BC bisecare.

Quaerenda sunt duo puncta eodem modo se habentia ad B et C. Quod fiet si duo triangula isoscelia BAC, BDC qualiacunque construantur super basi BC, recta per angulos basi oppositos seu per eorum apices ducta, basin bisecabit.

Schol. Euclides pro prop. 8. et 9. utitur triangulo aequilatero, sed praestat adhibere constructionem magis generalem.

Proposition 9.[25] To bisect a given line BC. [See Figure 9.]

Two points should be sought relating in the same way to B and C. This will happen if two isosceles triangles BAC, BDC of any sort are constructed over the base BC; the line drawn through the angles opposite the base, or through their apices, will bisect the base.

Scholium. Euclid uses an equilateral triangle for Propositions 8 and 9, but it is better to apply a more general construction.

[25]The manuscript has “Prop. 8” which repeats the number 8. It is not clear why, although Prop. 7 in Book 1 of Euclid is missing from Leibniz’s presentation, so Leibniz’s numbering does not align with Euclid at Props. 7, 8, 9 above. It is possible that Leibniz considered his Prop. 8 to incorporate both of Euclid’s Props. 7 and 8.