G. W. Leibniz

Initia Rerum Mathematicarum Metaphysica

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Gerhardt (ed.), *Leibnizens mathematische Schriften*, Bd. VII (1863), Nr. III, S. 17–29. Latin text collated against the scan of the print.

Initia Rerum Mathematicarum Metaphysica

Cum insignis Mathematicus Christianus Wolfius nuper in Cursu suo Mathematico Latino meditationes quasdam meas circa Analysin Axiomatum et circa naturam similitudinis attigerit et pro more suo illustrarit (vid. Act. Erudit. A. 1714), visum est nonnulla huc spectantia, dudum a me animo concepta, ne intercidant proferre, ex quibus intelligi potest, esse artem quandam Analyticam Mathematica ampliorem, ex qua Mathematica scientia pulcherrimas quasque suas Methodos mutuatur. Paulo ergo altius ordiri placet:

Si plures ponantur existere rerum status, nihil oppositum involventes, dicentur existere simul. Itaque quae anno praeterito et praesente facta sunt negamus esse simul, involvunt enim oppositos ejusdem rei status.

Si eorum quae non sunt simul unum rationem alterius involvat, illud prius, hoc posterius habetur. Status meus prior rationem involvit, ut posterior existat. Et cum status meus prior, ob omnium rerum connexionem, etiam statum aliarum rerum priorem involvat, hinc status meus prior etiam rationem involvit status posterioris aliarum rerum atque adeo et aliarum rerum statu est prior. *Et ideo quicquid existit alteri existenti aut simul est aut prius aut posterius.*

Initia Rerum Mathematicarum Metaphysica

Since the distinguished mathematician Christian Wolff, in his Latin Course in Mathematics, recently touched on and illustrated, in his own way, certain meditations of mine about the Analysis of Axioms and about the nature of similarity (see Act. Erudit A. 1714), it seemed good to set out some things pertaining to this that I worked out mentally some time ago, lest they be lost; from which it can be understood that there is a certain Analytical art broader than Mathematics, from which Mathematical science borrows certain of its most beautiful Methods. I would like to start, therefore, a little deeper:

*If one supposes that multiple states of things exist, involving nothing opposed, they are said to exist **simultaneously**.* And so we say that what happened in the past year and the present year are not simultaneous, for they involve opposed states of the same thing.

*If one of those that are not simultaneous involves the reason for another, the former is considered **prior** and the latter **posterior**.* My prior state involves the reason that the posterior exists. And since my prior state, because of the connection of all things, also involves the prior state of other things, hence my prior state also involves the reason for the posterior state of other things and so also is prior to the state of other things. And so, *whatever exists is either simultaneous with, or prior to, or posterior to, another existing thing*.

Tempus est ordo existendi eorum quae non sunt simul. Atque adeo est ordo mutationum generalis, ubi mutationum species non spectatur.

Duratio est temporis magnitudo. Si temporis magnitudo aequabiliter continue minuatur, tempus abit in *Momentum*, cujus magnitudo nulla est.

Spatium est ordo coexistendi seu ordo existendi inter ea quae sunt simul.

Secundum utrumque ordinem (temporis vel spatii) propiora sibi aut remotiora censentur, prout ad ordinem inter ipsa intelligendi plura paucioraque correquiruntur. Hinc duo puncta propiora sunt, quorum interposita ex ipsis maxime determinata dant aliquid simplicius. — Tale interpositum maxime determinatum, est via ab uno ad aliud simplicissima, minima simul et maxime aequabilis, nempe recta, quae minor interjecta est inter puncta propiora.

Extensio est spatii magnitudo. Male Extensionem vulgo ipsi extenso confundunt, et instar substantiae considerant.

Si spatii magnitudo aequabiliter continue minuatur, abit in punctum cujus magnitudo nulla est.

Situs est coexistentiae modus. Itaque non tantum quantitatem, sed et qualitatem involvit.

Quantitas seu Magnitudo est, quod in rebus sola compraesentia (seu perceptione simultanea) cognosci potest. Sic non potest cognosci, quid sit pes, quid ulna, nisi actu habeamus

Time *is the order of existing of those that are not simultaneous.* And so it is also the general order of changes when the species of the changes is not in view.

Duration *is the magnitude of time.* If the magnitude of time is diminished continuously and uniformly [aequabiliter], time vanishes in a Moment, which has no magnitude.

Space *is the order of coexisting*, or the order of existing among things that are simultaneous.

According to either order (of time or of space) things are deemed ***closer together or farther apart*** *according as more or fewer things are co-required to understand the order between them.* Hence two points are closer whose interpolating [points] maximally determined by them give something simpler.[1] Such a maximally determined interpolating thing is the simplest path from the one to the other, minimal and at the same time maximally uniform [aequabilis], namely a line, of which a lesser one intervenes between closer points.

Extension *is the magnitude of space.* Frequently extension is wrongly conflated with the extensum itself, and considered in the likeness of a substance.

If the magnitude of space is diminished uniformly and continuously, it vanishes in a point which has no magnitude.

Situs *is the mode of coexistence.* Thus it involves not only quantity, but also quality.

Quantity *or Magnitude is that which can be known in things only by co-presence (or simultaneous perception).* Thus it cannot be known what a foot or a cubit is, unless we actually have something like a

[1] Cf. Leibniz's definition of the line determined by two points as the set of points which are unique of their situs to the two given points.

aliquid tanquam mensuram, quod deinde aliis applicari possit. Neque adeo, pes ulla definitione satis explicari potest, nempe quae non rursus aliquid tale involvat. Nam etsi pedem dicamus esse duodecim pollicum, eadem est de pollice quaestio, nec majorem inde lucem acquirimus, nec dici potest, pollicis an pedis notio sit natura prior, cum in arbitrio existat utrum pro basi sumere velimus.

Qualitas autem est, quod in rebus cognosci potest cum singulatim observantur, neque opus est compraesentia. Talia sunt attributa quae explicantur definitione aut per varias modificationes quas involvunt.

Aequalia sunt ejusdem quantitatis.

Similia sunt ejusdem qualitatis. Hinc si duo similia sunt diversa, non nisi per compraesentiam distingui possunt.

Hinc patet exempli causa, duo Triangula aequiangula habere latera proportionalia vel vicissim. — Nam sint latera proportionalia, similia utique sunt triangula, cum simili modo determinentur. Porro in omni triangulo summa angulorum est eadem, cum aequetur duobus rectis; ergo necesse est in uno rationem angulorum respondentium ad summam esse quae in altero; alioqui unum triangulum ab alio eo ipso distingui posset, ex se scilicet seu singulatim spectatum. Ita facile demonstratur quod alias per multos ambages.

Homogenea sunt quibus dari possunt aequalia similia inter se. Sunto A et B, et possit sumi L aequale ipsi A, et M aequale ipsi B sic ut L et M sint similia, tunc A et B appellabuntur Homogenea.

Hinc etiam dicere soleo, Homogenea esse quae per transformationem sibi reddi possunt similia, ut curva rectae. Nempe

measure that can then be applied to other things. Nor indeed can a foot be sufficiently explained by any definition which does not in fact involve again some such thing. For even if we said that a foot is twelve inches, there is the same question about an inch, and we do not gain any further insight from it, nor can one say whether the notion of an inch or a foot is prior in nature, since it is arbitrary which one we want to take as the basis.

*But **Quality** is that which can be known in things when they are observed individually* and co-presence is not needed. Such are attributes that are explained by a definition or through the various modifications that they involve.

Equals are of the same quantity.

Similars are of the same quality. Hence if two similars are distinct, they cannot be distinguished except by co-presence.

Hence it is clear for example that two equiangular Triangles have proportional sides and vice versa. For if the sides are proportional, the triangles are certainly similar, since they are determined in a similar manner. Further, in every triangle the sum of the angles is the same, since it is equal to two right angles; therefore it is necessary that the ratio of the corresponding angles to the sum in the one is what it is in the other; otherwise by that very fact the one triangle could be distinguished from the other, indeed viewed in itself or individually. Thus is easily demonstrated what otherwise requires many convolutions.

Homogeneous things are those to which equals can be given that are similar to each other. Let there be A and B, and if L can be taken equal to A and M equal to B such that L and M are similar, then A and B will be called Homogeneous.

Hence I also usually say that Homogeneous things are those that can be rendered similar by transformation, like a curve to a line. Namely si A transformetur in aequale sibi L, potest fieri simile ipsi B vel ipsi M, in quod transformari ponitur B.

Inesse alicui loco dicimus vel alicujus *ingrediens* esse, quod aliquo posito, eo ipso immediate poni intelligitur, ita scilicet ut nullis opus sit consequentiis. Sic ubi lineam aliquam finitam ponimus, ejus extrema ponimus ejus partes.

Quod inest homogeneum, *Pars* appellatur, et cui inest appellatur *Totum*, seu pars est ingrediens homogeneum.

Terminus communis est, quod duobus inest partem communem non habentibus. Quae quoties intelligantur partes ejusdem Totius, is terminus communis dicetur *Sectio* totius.

Hinc patet, Terminum non esse homogeneum terminato, nec Sectionem esse homogeneam secto.

Tempus et Momentum, Spatium et Punctum, Terminus et Terminatum, etsi non sint Homogenea, sunt tamen homogona, dum unum in alterum continua mutatione abire potest.

Locum qui alteri loco *inesse* dicitur, Homogonum intelligimus, quod si ejus sit pars aut parti aequalis, non tantum homogonus sed et homogeneus erit. Angulus etsi ad punctum sit, non tamen est in puncto, alioqui in puncto magnitudo intelligeretur.

Si pars unius sit aequalis alteri toti, illud vocatur Minus, hoc Majus.

Itaque Totum est majus parte. Sit totum A, pars B, dico A esse majus quam B, quia pars ipsius A (nempe B) aequatur toti B. Res etiam Syllogismo exponi potest, cujus Major propositio

if A is transformed into its equal L, it can be made similar to B or to M into which B is assumed to be transformed.

That is said to *be-in* some locus or to be an *ingredient* of some thing, which, when the [locus or] thing is posited, is understood to be posited immediately by that very fact, so that, indeed, there is no need for inferences. Thus when we posit some finite line, we posit its extrema, its parts.

Something homogeneous that is-in, is called a *Part*, and that which it is-in is called the *Whole*, or i.e. a part is a homogeneous ingredient.

A *common boundary* is that which is-in two things not having a common part. Whenever these are understood to be parts of the same whole, that common boundary is called a *Section* of the whole.

Hence it is clear that a Boundary is not homogeneous with what is bounded, nor is a Section homogeneous with what is sectioned.

Time and Moment, Space and Point, Boundary and Bounded, even though they are not Homogeneous, are nevertheless ***homogonous***, *in that one can pass into the other by a continuous change*.

A *locus* which is said to *be-in* another locus, we understand to be Homogonous, because if it were a part or equal to a part of it, it would be not only homogonous but also homogeneous. Even though an angle is at a point, it is not in a point, otherwise magnitude would be thought to be in a point.

If a part of one is equal to another whole, the latter is called *Lesser*, the former *Greater*.

And so *the Whole is greater than a part*. Let A be the whole, B a part; I say that A is greater than B, because a part of A (namely B) is equal to the whole B. The matter can also be set out in a Syllogism, whose est definitio, Minor propositio est identica:

Major proposition is a definition and Minor proposition is identical [identica]:

Quicquid ipsius A parti aequale est, id ipso A minus est, ex definitione,

Whatever is equal to a part of A, that is less than A, by definition;

B est aequale parti ipsius A, nempe sibi, ex hypothesi,

B is equal to a part of A, namely itself, by hypothesis;

ergo B est minus ipso A.

Therefore B is less than A.

Unde videmus demonstrationes ultimum resolvi in duo indemonstrabilia: Definitiones seu ideas, et propositiones primitivas, nempe identicas, qualis haec est B est B, unumquodque sibi ipsi aequale est, aliaeque hujusmodi infinitae.

Hence we see that demonstrations ultimately are resolved into two indemonstrables: Definitions or ideas, and primitive, namely identical, propositions, such as this is: B is B, each thing is equal to itself, and infinite others of this kind.

Motus est mutatio situs.

Motion is change of situs.

Movetur, in quo est mutatio situs, et simul ratio mutationis.

That *moves* in which there is a change of situs and simultaneously the reason for the change.

Mobile est homogonum extenso, nam et punctum mobile intelligitur.

A movable thing is homogonous with an extensum, for even a point is understood to be movable.

Via est locus continuus successivus rei mobilis.

A *path* is the continuous successive locus of a movable thing.

Vestigium est locus rei mobilis, quem aliquo momento occupat. Hinc vestigium termini est sectio viae quam terminus describit, cum scilicet mobile per sua vestigia non incedit.

A *track* is the locus of a movable thing which it occupies at some moment. Hence the track of a boundary is a section of the path which the boundary describes, when of course the movable thing does not proceed in its own tracks.

Mobile per sua vestigia incedere dicitur, cum quodvis ejus punctum extra terminum in locum alterius puncti ejusdem mobilis continuo succedit.

A *movable* is said to *proceed in its own tracks* when any point of it except the boundary continuously follows in the place of another point of the same movable thing.

Quodsi Mobile sic moveri non ponatur, tunc *Linea* est via puncti.

Superficies est via Lineae.

Amplum vel *Spatium* vel ut vulgo solidum est via superficiei.

Magnitudines viarum quibus punctum lineam, linea superficiem, superficies amplum describit, vocantur *longitudo*, *latitudo*, *profunditas*. Vocantur dimensiones, et in Geometria ostenditur non nisi tres dari.

Latitudinem habet, cujus datur sectio extensa, seu quod extenso terminatur.

Profunditatem habet, quod extensum non terminat, seu quod sectio extensi esse non potest, in profundo scilicet est plus aliquid quam quod terminus esse possit.

Linea est ultimum terminans extensum.

Amplum est ultimum Terminatum extensum.

Similitudo vel dissimilitudo in amplo seu spatio cognoscitur ratione terminorum, itaque *amplum*, cum plus aliquid sit quam quod terminus esse possit, intus ubique simile est. Amplaque quorum omnimodae extremitates coincidunt, congruunt, assimilantur, sunt coincidentia, congruentia, similia. Idem est in plano, quod est superficies intus uniformis vel sibi similis, et in *recta*, quae est linea intus sibi similis.

Omnimoda extremitas in Extensis latitudinem habentibus

But if a Movable is not assumed to move in this way, then[2] A *Curve* is the path of a point.

A *Surface* is the path of a Curve.

An *Expanse* [Amplum] or *Space*, or in common terms a solid, is the path of a surface.

The magnitudes of paths by which a point describes a curve, a curve a surface, and a surface an expanse, are called *length*, *width*, and *depth*. They are called dimensions, and in Geometry it is shown that there are only three.

That *has Width* of which there is an extended section, or which is bounded by something extended.

That *has Depth* which does not bound an extensum, or which cannot be the section of an extensum; indeed, in depth there is something more than could be a boundary.

A curve is the last bounding extensum.

An expanse is the last Bounded extensum.

Similarity or dissimilarity in an expanse or space is recognized based on the boundaries, and so an *expanse*, since it is something more than could be a boundary, is everywhere similar inside. Expanses whose entire extremities coincide, are congruent, are similar, will be coincident, congruent, similar. It is the same in the plane, which is a surface uniform or similar to itself inside, and in a *line*, which is a curve similar to itself inside.

The *entire extremity* in Extensa having width can be called the *Ambit*.

[2]A marginal insertion ends here abruptly, with space below. A connecting line indicates it is somewhat likely that Leibniz intended the text to continue directly with the next sentence.

Ambitus appellari potest. Sic ambitus circuli est peripheria, ambitus sphaerae est superficies sphaerica.

Punctum (spatii scilicet) est locus simplicissimus, seu locus nullius alterius loci.

Spatium absolutum est locus plenissimus seu locus omnium locorum.

Ex uno puncto nihil prosultat.

Ex duobus punctis prosultat aliquid novi, nempe punctum quodvis sui ad ea situs unicum, horumque omnium locus, id est *recta* quae per duo puncta proposita transit.

Ex tribus punctis prosultat *planum*, id est locus omnium punctorum sui ad tria puncta non in eandem rectam cadentia situs unicorum.

Ex quatuor punctis non in idem planum cadentibus prosultat *Spatium absolutum*. Nam quodvis punctum sui ad quatuor puncta in idem planum non cadentia situs unicum est.

Prosultandi vocabulo utor ad ideam indicandam novam, dum ex quibusdam positis aliquid aliud determinatur eo ipso quod suae ad ipsa relationis unicum est. — Relatio autem hic intelligitur situs.

Tempus in infinitum continuari potest. Cum enim totum tempus sit simile parti, habebit se ad aliud tempus, ut pars se habet ad ipsum, et ita in alio majore tempore continuari intelligitur.

Thus the ambit of a circle is the circumference, the ambit of a sphere is the spherical surface.

A *point* (that is, of space) is the simplest locus, or a locus of no other locus.

Absolute space is the fullest locus, or the locus of all loci.

From one point nothing prosults.[3]

From two points something new prosults, namely any point unique of its situs to them, and the locus of all these, that is, the line which passes through the two proposed points.

From three points a *plane* prosults, that is, the locus of all points unique of their situs to three points not falling on the same line.

From four points not falling on the same plane prosults *absolute Space*. For every point is unique of its situs to four points not falling in the same plane.

I use the word *prosulting* to indicate a new idea, when from certain posited things something else is determined, by that very fact, which is unique of its relation to them. By relation here we understand situs.

Time can be continued to infinity. For since a whole of time is similar to a part, it relates to another time as the part relates to it, and so it is understood to continue in another greater time.

[3] The Latin *prosulto* may be Leibniz's coinage, or a technical usage he invents for a very rare word.

Similiter et *spatium solidum seu amplitudo continuari in infinitum potest*, quandoquidem ejus pars sumi potest similis toti. Et hinc planum quoque et recta continuantur in infinitum. Eodem modo ostenditur, spatium velut rectam, itemque tempus, et in universum continuum in infinitum subdividi posse. Nam in recta et in tempore pars est similis toti atque adeo in eadem ratione secari potest, qua totum, et licet sint extensa, in quibus pars non est similis toti, possunt tamen transformari in talia, et in eadem ratione secari, in qua ea, in quae transformantur.

Sequitur etiam ex his, *quovis* motu posse assumi celeriorem et tardiorem in data ratione: radio enim rigido circa centrum acto motus punctorum sunt ut distantiae eorum a centro, itaque celeritates variari possunt ut rectae.

Aestimatio magnitudinum duplex est, imperfecta et perfecta; imperfecta, cum aliquid majus minusve altero dicimus, quamvis non sint homogenea, nec habeant proportionem inter se, quemadmodum si quis diceret, Lineam esse majorem puncto, aut superficiem linea. Et tali modo Euclides dixit, Angulum contactus esse minorem quovis rectilineo, etsi revera nulla inter hos toto genere diversos sit comparatio, quin nec homogenea sunt, nec continua mutatione ab uno in alterum transiri potest. In aestimationibus perfectis inter homogenea obtinet haec regula, ut transeundo continue ab uno extremo ad aliud, transeatur per omnia intermedia; sed non obtinet in imperfectis, quia quod medium dicitur heterogeneum est, itaque transeundo continue ab angulo acuto dato ad angulum rectum non transitur per Angulum semicirculi seu radii ad circumferentiam, etsi is angulo recto minor, et quovis acuto major dicatur, id Majus enim hic sumitur improprie pro eo quod intra alterum cadit.

Similarly also *solid space or expanse can be continued to infinity*, inasmuch as a part of it can be taken similar to the whole. And hence a plane and also a line continue to infinity. In the same way one shows that space, just like a line and also time, and the continuum in general, can be subdivided to infinity. For in a line and in time a part is similar to the whole and so can be sectioned in the same ratio as the whole, and even if there are extensa in which a part is not similar to the whole, still they can be transformed into such, and sectioned in the same ratio as those into which they were transformed.

It also follows from these things that for *any* movement one can assume a faster and slower one in a given ratio: indeed when a rigid radius is driven around a center, the movements of the points are as their distances from the center, and so the speeds may be varied as the lines.

The *estimation* of magnitudes is twofold, imperfect and perfect; imperfect, when we say that something is greater than or less than something else, although they are neither homogeneous nor have a proportion to each other, as when someone says that a Curve is greater than a point, or a surface than a curve. In this sense Euclid said that an Angle of contact is less than any rectilinear angle, even though in reality there is no comparison between these completely distinct kinds, since they are not homogeneous, nor is it possible to pass from the one to the other by a continuous change. In perfect estimations between homogeneous things the following rule obtains, that in passing continuously from one extreme to another, one passes through all intermediates; but this does not obtain in imperfect estimations because what is called a mediate is heterogeneous, and so, passing from a given acute angle to a right angle, one does not pass through the Angles of a semicircle, or radii to the circumference, even though it is said to be less than a right angle and greater than any acute angle, for here the Greater is taken improperly for that which falls inside another.

Plures secundum quantitatem dantur relationes; sic duae rectae possunt eam habere Relationem inter se, ut summa earum aequetur constanti rectae. Et infinita possunt dari paria rectarum hanc relationem inter se habentia, nempe x et y, ita ut sit $x + y = a$, si verb. gr. a sit ut 10, possunt x et y esse ut 1 et 9, ut 2 et 8, ut 3 et 7, ut 4 et 6, ut 5 et 5, ut 6 et 4, ut 7 et 3, ut 8 et 2, ut 9 et 1. Sed possunt etiam infiniti sumi fracti infra 10, qui satisfaciunt. Sic datur relatio talis inter duas rectas x et y, ut quadrata earum simul sumta aequentur quadrato dato rectae a, ita fiet $xx + yy = aa$: et talium etiam dari possunt paria infinita, et haec est in Circulo relatio sinus complementi ad sinum vel contra, nam uno posito x, alter est y, radius autem est a. Et tales relationes possunt fingi infinitae, tot quot species linearum in plano describi possunt. Veluti si x sint abscissae ex recta directrice, y erunt ordinatae inter se parallelae ad abscissas applicatae, quae terminantur in Linea.

Sed omnium Relationum simplicissima est, quae dicitur *Ratio* vel *Proportio*, eaque est Relatio duarum quantitatum homogenearum, quae ex ipsis solis oritur sine tertio homogeneo assumto. Veluti si sit y ad x ut numerus ad unitatem seu $y = nx$, quo casu x positis abscissis, y ordinatis, locus est recta, locus inquam seu Linea quam ordinatae terminantur. Ex quo etiam patet, si esset aequatio localis cujuscunque gradus velut $lx^3 + my^3 + nxxy + pxyy = 0$, ubi l, m, n, p sint meri numeri, locum ad quem sit aequatio fore rectam et datam esse rationem ipsarum x et y.

Sint datae duae rectae, quae inter se comparentur utcunque. Verb. gr. detrahatur minor ex majore, quoties fieri potest, et residuum rursus ex minore, et ita porro residuum ex illo quod quoties fieri potest est detractum, donec vel sequatur

There are various relations according to quantity. Thus two lines can have the Relation to each other that their sum is equal to a constant line. And infinitely many pairs of lines can be given having this relation to each other, namely x and y such that $x + y = a$, if for instance a is as 10, then x and y can be as 1 and 9, as 2 and 8, as 3 and 7, as 4 and 6, as 5 and 5, as 6 and 4, as 7 and 3, as 8 and 2, as 9 and 1. But infinitely many fractions can also be taken under 10 that satisfy it. Thus too, such a relation is given between two lines x and y that their squares taken together equal the given square of the line a, so we have $xx + yy = aa$; and infinitely many pairs of such lines can also be given, and this is the relation of the cosine [sinus complementi] to the sine in the Circle and conversely, indeed setting the one as x, the other is y and the radius is a. And infinitely many such relations can be created, as many as the species of curves that can be described in the plane. For instance, if x are the abscissae of the directrix line, y will be the mutually parallel ordinates attached to the abscissae which end at the Curve.

But the simplest of all Relations is that which is called *Ratio* or *Proportion*, and it is the Relation of two homogeneous quantities that arises from them alone without assuming a third homogeneous one. For instance when y is to x as a number is to unity, or $y = nx$, in which case, setting x as the abscissae and y as the ordinates, the locus is a line; I mean the locus or Curve at[4] which the ordinates terminate. It is also clear from this that, if the equation were a local one, of whatever degree, such as $lx^3 + my^3 + nxxy + pxyy$ where l, m, n, p are pure numbers, then the locus for which this is the equation would be a line, and the ratio of x and y would be given.

Let two lines be given which are compared with each other in some manner. For example let the lesser be subtracted from the greater as many times as possible, and the remainder again from the lesser, and then again the remainder [subtracted] from that which is sub-

[4]The Latin here is not really legible, but is plausibly 'ad quam'.

exhaustio, ultimo subtrahendo existente communi Mensura, si quantitates sunt commensurabiles, vel habeatur lex progressionis in infinitum, si sint incommensurabiles. Et eadem erit series numerorum Quotientium, cum eadem est proportio. Nempe si sit a ad b ut

$$l+\cfrac{1}{m+\cfrac{1}{n+\cfrac{1}{p+etc.}}}$$

ad unitatem, erit l, m, n, p etc. series numerorum quotientium. — Verb. gr. si a sit 17 et b sit 5, series tantum constabit ex tribus l, m, n, qui numeri erunt 3, 2, 2. Si a et b sint partes rectae extrema et media ratione sectae, erit a major ad b minorem, ut $1+\frac{1}{1+\frac{1}{1+\frac{1}{1+etc.}}}$ ad unitatem; quotientes erunt unitates, et series eorum ibit in infinitum. — Sic a et b quaecunque rectae erunt ad se invicem ut $\frac{l}{1}+\frac{m}{2}+\frac{n}{4}+\frac{p}{8}+\frac{q}{16}+$etc. ad 1, posito l, m, n, p, q etc. esse 0 vel 1, quae series vel finitur vel periodica est, cum numeri sunt commensurabiles.

Ex his sequitur, lineas similes esse in ratione rectarum Homologarum, superficies similes ut rectarum homologarum quadrata, solida similia ut eorum cubos. Sint duo Extensa similia A et L, et homologa homogenea et B et M. Quia A cum B seu A; B simile est ipsi L cum M seu L; M, erit ratio A ad B eadem quae L ad M. alioqui ipsum A; L ab ipso B; M aliter quam per compraesentiam distingui posset; prodibunt enim alii numeri rationem exprimentes. Ergo permutando erit A ad L ut B ad M, ut ponebatur. Sic ostendetur circulos esse ut quadrata diametrorum, sphaeras ut cubos diametrorum. Circuli (sphaerae) erunt A et L, quadrata homologa (cubi homologi) B et M.

tracted as many times as possible, until either exhaustion is attained, the last subtrahend being a common Measure, if the quantities are commensurable, or else one has a law of progression to infinity if they are incommensurable. And the series of Quotient numbers will be the same when the proportion is the same. Namely, if a is to b as

$$l+\cfrac{1}{m+\cfrac{1}{n+\cfrac{1}{p+etc.}}}$$

is to unity, then l, m, n, p, etc. will be the series of quotient numbers. For example, if a is 17 and b is 5, the series will only consist of three, l, m, n, which numbers are 3, 2, 2.[5] If a and b are parts of a line sectioned with an extreme and middle ratio, the greater a will be to the lesser b as $1+\frac{1}{1+\frac{1}{1+\frac{1}{1+etc.}}}$ to unity; the quotients will be units, and their series will go on to infinity. Thus any lines a and b will be to each other as $\frac{l}{1}+\frac{m}{2}+\frac{n}{4}+\frac{p}{8}+\frac{q}{16}+$ etc. to 1, setting l, m, n, p, q to be 0 or 1, and this series either terminates or is periodic when the numbers are commensurable.

It follows from this that similar curves are in the ratio of Homologous lines, similar surfaces are as the squares of homologous lines, similar solids are as their cubes. Let there be two similar Extensa A and L, and homologous, homogeneous things B and M. Since A together with B, or $A\,;B$,[6] is similar to L together with M, or $L\,;M$, the ratio of A to B will be the same as that of L to M, otherwise $A\,;L$ could be distinguished from $B;M$ in some way other than by copresence; indeed different numbers expressing the ratios would result. Therefore, by permuting, A will be to L as B to M, as was proposed. Thus it is shown that circles are as the squares of the diameters, spheres as the cubes of the diameters. The circles (spheres) will be A and L, the homologous squares (homologous cubes) will be B and M.

[5] The MS has 2, 3, 2, presumably by mistake.

[6] The MS here is not legible.

Ex his manifestum est, *Numerum* in genere integrum, fractum, rationalem, surdum, ordinalium, transcendentem generali notione definiri posse, ut sit id quod homogeneum est Unitati, seu quod se habet ad Unitatem, ut recta ad rectam. — Manifestum est etiam, si Ratio a ad b consideretur ut numerus qui sit ad Unitatem, ut recta a ad rectam b, fore Rationem ipsam homogeneam Unitati; Unitatem autem repraesentare Rationem aequalitatis.

From this it is clear that *Number* in general, whole, fractional, rational, surd, ordinal, transcendental, can be defined by the general notion that it is what is homogeneous to Unity, or that which relates to Unity as a line to a line. It is also clear that, if the Ratio of a to b were considered as the number which is to Unity as the line a to the line b, then the Ratio itself would be homogeneous to Unity, and Unity, moreover, represents the Ratio of equality.

Notandum est etiam, totam doctrinam Algebraicam esse applicationem ad quantitates Artis Combinatoriae, seu doctrinae de Formis abstractae animo, quae est Characteristica in universum, et ad Metaphysicam pertinet. Sic productum multiplicatione $a+b+c+etc.$ per $l+m+n+etc.$ nihil aliud est quam summa omnium binionum ex diversi ordinis literis, et productum ex tribus ordinibus invicem ductis, $a+b+c+etc.$ in $l+m+n+etc.$ in $s+t+v+etc.$ fore summam omnium ternionum ex diversi ordinis literis; et ex aliis operationibus aliae prodeunt formae.

It should also be noted that the whole doctrine of Algebra is an application to quantities of the Combinatorial Art, or of the abstract doctrine of Forms in the mind, which is Characteristic in general and pertains to Metaphysics. Thus the product of the multiplication of $a+b+c+etc.$ by $l+m+n+etc.$ is nothing but the sum of all pairs of letters from different series [ordo], and the product of three series multiplied together, $a+b+c+etc.$ by $l+m+n+etc.$ by $s+t+v+etc.$, will be the sum of all triples of letters from different series; and from other operations other forms result.

Hinc in calculo non tantum lex homogeneorum, sed et justitiae utiliter observatur, ut quae eodem modo se habent in datis vel assumtis, etiam eodem modo se habeant in quaesitis vel provenientibus, et qua commode licet inter operandum eodem modo tractentur; et generaliter judicandum est, datis ordinate procedentibus etiam quaesita procedere ordinate. Hinc etiam sequitur *Lex Continuitatis* a me primum prolata, qua fit ut lex quiescentium sit quasi species legis in motu existentium, lex aequalium quasi species legis inaequalium, ut lex Curvilineorum est quasi species legis rectilineorum, quod semper locum habet, quoties genus in quasi-speciem oppositam desinit. Et hic pertinet illa ratiocinatio quam Geometrae dudum admirati sunt, qua ex eo quod quid ponitur esse, directe probatur id non esse, vel contra, vel qua quod

Hence in calculus it is useful to observe not only the law of homogeneous things, but also that of justice, that whatever things behave in the same way in the givens or assumptions, also behave in the same way in what is sought or what emerges, and which allows one to treat them conveniently in the same way while performing operations;[7] and in general one may conclude that when the givens come in order, so also the things sought come in order. Hence also follows the *Law of Continuity* which I was the first to set forth, by which it happens that the law of things at rest is like [quasi] a species of the law of things being in motion, the law of equals like a species of the law of unequals, as the law of Curvilinear things is like a species of the law of rectilinear things. This always takes place whenever a genus ends in an opposite quasi-species. And here pertains that argument Geometers used to marvel at by which, from that which is assumed to be something, one

[7]Leibniz inserted the last clause after writing the subsequent sentence, and its syntax is not entirely compatible with that of the first part.

velut species assumitur, oppositum seu disparatum reperitur. Idque continui privilegium est; *Continuitas* autem in tempore, extensione, qualitatibus, motibus, omnique *naturae transitu* reperitur, qui nunquam fit per saltum.

directly proves that it is not or conversely, or by which, something taken as if a species, is found to be opposite or disparate. And that is the privilege of the continuous; *Continuity* can in fact be found in time, extension, qualities, motions, and every *transition of nature*, which never happens by a jump.

Situs quaedam coexistendi relatio est inter plura, eaque cognoscitur per alia coexistentia, intermedia, id est quae ad priora simpliciorem habent coexistendi relationem.

Situs is a certain relation of coexisting among multiple things, and it is known through other coexisting things, intermediates, that is, things which have a simpler relation to the former ones of coexisting.

Coexistere autem cognoscimus non ea tantum quae simul percipiuntur, sed etiam quae successive percipimus, modo ponatur durante transitu a perceptione unius ad perceptionem alterius aut non interiisse prius, aut non natum esse posterius. Ex illa hypothesi sequitur nunc ambo coexistere, cum posterius attigimus; ex hac sequitur ambo extitisse cum prius disereremus.

We know to coexist not only things which are perceived simultaneously, but also things which we perceive successively, if it be assumed only that, during the transition from perception of the one to perception of the other, either the earlier has not perished, or the later has not been born. From the first hypothesis it follows that both coexist now when we reach the later; from the second it follows that both existed when we left the earlier.

Est autem in percipiendi transitu quidam ordo, dum ab uno ad aliud per alia transitur. Atque hoc *via* dici potest. Sed hic ordo cum variari possit infinitis modis, necesse est unum esse simplicissimum, qui scilicet sit secundum ipsam rei naturam procedendo per determinata intermedia, id est per ea quae se habent ad utrumque extremum quam simplicissime. Id enim nisi esset, nullus esset ordo, nulla discernendi ratio in coexistentia rerum, cum a dato ad datum per quodvis iri possit. Atque haec est via minima ab uno ad aliud, cujus magnitudo *distantia* appellatur.

Now there is a certain order in the transition of perception, when one passes from one to another through other things. And this can be called a *path*. But as this order can vary in infinitely many ways, it is necessary that one is the simplest, which of course is by proceeding through the determined intermediates according to the very nature of the thing, that is, through those which relate to either extreme most simply. For if it did not exist, there would be no order, no basis for distinguishing [ratio discernendi] in the coexistence of things, since one could go from given to given through anything. And this is the minimal path from one to another, whose magnitude is called *distance*.

Haec ut melius intelligantur, nunc quidem abstrahemus animum ab his quae in singulis spectari possunt, de quorum distantia agitur: ita considerabimus ea, tanquam in singulis plura spectanda non essent, seu considerabimus ea tanquam Puncta. Nam *punctum* est, in quo nihil aliud ei coexistens ponitur, ita ut quicquid in ipso est, ipsum sit.

In order to understand these things better, we shall now abstract our mind from what can be seen in the individual things whose distance is being treated; thus we will consider them as if a plurality is not to be seen in the individuals, i.e. we will consider them as Points. For something is a *point* in which nothing else coexisting with it is posited, so that whatever is in it, is it.

Ita via puncti erit *linea*, quae utique *latitudinem* non habebit, quia sectio ejus quae in puncto fit longitudinem non habet.

Ex uno puncto dato nihil determinatur praeterea. Sed duobus datis punctis determinatur via ab uno ad aliud simplicissima, quam appellamus *Rectam*.

(1) Hinc sequitur *primo*, rectam esse minimam a puncto ad punctum, seu magnitudinem ejus esse punctorum distantiam.

(2) *Secundo*, rectam esse inter sua extrema aequabilem. Neque enim aliquid assumitur, unde reddi possit ratio varietatis.

(3) Itaque oportet, ut unus locus puncti in ea moti ab altero discerni non possit seposito respectu ad extrema. Hinc et pars rectae recta est, itaque intus ubique sibi similis est, nec duae partes discerni possunt inter se, cum suis extremis discerni non possint.

(4) Sequitur etiam, extremis positis similibus aut congruis aut coincidentibus, ipsas rectas similes, congruas aut coincidentes esse. Extrema autem semper sunt similia. Itaque rectae duae quaevis sunt similes, et pars etiam toti.

(5) *Tertio* ex definitione sequitur, procedere rectam per puncta suae relationis unica ad duo puncta data, quae ratio est maxime determinata. Oportet autem talia dari, alioqui ex duobus datis nihil resultaret novi determinati. Et si daretur aliud punctum eodem modo se habens ad A et B simul sumta ut propositum, nulla esset ratio cur via illa simplicissima determinata per unum potius quam per aliud procederet. Patet

Then the path of a point will be a *curve*, which of course does not have *width* because a section of it, which happens at a point, does not have length.

From one given point nothing else is determined. But with two points given, the simplest path from the one to the other is determined, which we call a *Line*.

(1) Hence it follows *first* that a line is the minimal [path] from point to point, i.e. its magnitude is the distance of the points.

(2) *Second,* that a line is uniform [aequabilem] between its extremes. For nothing is assumed from which a reason for a variation could be provided.

(3) And so it must be the case that one locus of a point moving along it cannot be distinguished from another, setting aside reference to the extremes. Hence also a part of a line is a line, and so inside it is self-similar everywhere, and neither can two parts be distinguished from each other when their extremes cannot be distinguished.

(4) It also follows that, supposing the extremes are similar or congruent or coincident, the lines themselves are similar, congruent, or coincident. But the extremes are always similar. And so any two lines are similar, and also a part to the whole.

(5) *Third,* it follows from the definition that a line proceeds through points unique of their relation to the two given points, which rule [ratio] is maximally determined. There must be such [points], otherwise no new determined thing would result from two givens. And if there were some other point relating in the same way to A and B taken together, as proposed, there would be no reason why that simplest, determined path would proceed through one rather than the other. This is also

etiam hoc ex praecedente, quia dum ostendimus extremis datis rectam esse determinatam seu extremis coincidentibus coincidere rectas.

(6) *Quarto* sequitur, rectam in omnes plagas se habere eodem modo nec concavum habere et convexum ut curvam, cum ex duobus punctis assumtis A et B nulla ratio diversitatis reddi possit.

(7) Et proinde, si duo assumantur puncta quaecunque extra rectam L et M quae se eodem modo habeant ad duo puncta rectae collective, seu ita ut L se habeat ad A et B, ut M se habet ad A et B, etiam eodem modo se habebunt ad totam rectam, seu L se habebit ad rectam per A, B, ut M se ad eam habet.

(8) Manifestum etiam est, rectam *rigidam* seu cujus puncta non mutant situm inter se, duobus in ea punctis manentibus immotis moveri non posse, alioqui enim plura darentur puncta eodem modo se habentia ad duo puncta immota, nempe tam punctum in quo fuit punctum mobile, quam illud ad quod est translatum.

(9) Sequitur vicissim, alia omnia puncta, quae in rectam per A et B transeuntem non cadunt seu quae ipsis A et B non sunt in directum, mobilia esse situ ad A et B servato, ipsis A et B manentibus immotis, cum recta sit locus omnium punctorum unice se habentium ad A et B; caetera ergo variari possunt et quidem in omnes partes, cum recta eodem modo se habeat in omnes partes.

(10) Itaque si Extensum rigidum moveatur duobus punctis

clear from the preceding, when[8] we showed that the line is determined given the extremes, or the lines coincide if the extremes coincide.

(6) *Fourth,* it follows that a line behaves in the same way toward all regions,[9] neither can it be concave and convex like a curve, since from two assumed points A and B, no reason could be given for a distinction.

(7) And hence if any two points L and M are assumed outside the line, which relate in the same way to two points of the line collectively, or so that L relates to A and B as M relates to A and B, then they will also relate in the same way to the whole line, or L will relate to the line through A and B as M relates to it.

(7)[10] It is also clear that a *rigid* line, or one whose points do not change situs among themselves, cannot be moved while two points on it remain unmoved, for otherwise multiple points would be given relating in the same way to the two unmoved points, namely the point at which the movable point was, as well as that to which it is transferred.

(8) It follows in turn that all other points which do not fall on the line passing through A and B, i.e. which are not in alignment with A and B, are movable with the situs to A and B preserved, A and B remaining unmoved, since the line is the locus of all points relating uniquely to A and B; therefore any others can vary, and indeed to all sides, since the line relates in the same way to all sides.

(9) And so if a rigid Extensum moves with two points remaining un-

[8]The Latin here, ‘quia dum,’ has a superfluous conjunction.
[9]Leibniz deleted ‘partes’ and inserted ‘plagas’.
[10]The MS has two paragraphs numbered with (7). This second one has the number deleted and rewritten with a marginal note about the duplication.

manentibus immotis, omnia puncta ejus quiescentia cadent in rectam per puncta immota transeuntem, quodvis punctum mobile describet circulum circa eandem rectam velut axem.

Datis tribus punctis in eandem Rectam non cadentibus, quod inde determinatur est *planum*. Sint puncta A, B, C non cadentia in eandem rectam, ex A, B punctis determinatur recta per A et B, ex C et B punctis determinatur recta per C et B. Ex quovis puncto rectae per A et B, conjuncto cum quovis puncto rectae per C et B, nova determinatur recta, et proinde datis A, B, C, determinantur rectae infinitae quarum locus vocatur *planum*.

(1) Itaque *primo* planum est minimum inter sua extrema. Ambitus enim ejus non constat recta, quia recta spatium non claudit, alioqui pars rectae foret dissimilis toti. Ergo ambitu dato dantur tria puncta in eandem rectam non cadentia, ergo ex solo ambitu dato determinatur planum interceptum. Ergo est minimum.

(2) *Secundo* planum est aequabile inter sua extrema, quia nulla ratio varietatis ex hac origine ejus deduci potest.

(3) Unde sequitur, planum intus esse sibi simile, ita quod in eo movetur punctum, locum in quo est ab alio loco non distinguit nisi respectu ad extrema. Nec pars plani a parte nisi per extrema discerni potest.

(4) Sequitur et porro, plana quorum ambitus sunt similes aut congrui aut coincidentes, ipsa similia aut congrua aut coincidentia esse.

(5) *Tertio* ex definitione plani patet esse locum omnium punctorum ad tria data puncta unicorum.

moved, all its points at rest will fall on the line passing through the unmoved points, and any movable point will describe a circle around the same line as an axis.

Given three points not falling on the same Line, what is determined from this is a *plane*. Let A, B, C be points not falling on the same line; from the points A, B the line through A and B is determined, from the points C and B the line through C and B is determined. From any point of the line through A and B, together with any point of the line through C and B, a new line is determined, and hence given A, B, C, infinitely many lines are determined, the locus of which is called a *plane*.

(1) And so *first,* a plane is the minimum between its extremes. Indeed its ambit does not consist of a line, because a line does not enclose space, else a part of the line would be dissimilar to the whole. Therefore, the ambit being given, three points not falling on the same line are given, therefore from the given ambit alone the intervening [interceptum] plane is determined. Therefore it is the minimum.

(2) *Second,* a plane is uniform between its extremes, because no reason for variation can be deduced from this its origin.

(3) Whence it follows that a plane is self-similar inside, thus a point which moves in it does not distinguish the place where it is from another place except with respect to the extremes. Neither can a part of the plane be discerned from another part except through the extremes.

(4) It follows furthermore that planes whose ambits are similar or congruent or coincident are themselves similar or congruent or coincident.

(5) *Third,* it is clear from the definition of a plane that it is the locus of all points unique [of their relation] to three given points.

(6) *Quarto* sequitur planum utrinque se habere eodem modo, adeoque nec concavum habere neque convexum.

(7) Atque adeo puncto se habenti utcunque ad A, B, C simul vel ad planum ex his determinatum, respondens aliud punctum dari posse eodem modo se habens ad tria haec puncta simul sumta, cum nulla sit ratio diversitatis.

(8) Planum autem est latitudine praeditum, nam per lineam rectam secari potest, per bina data ejus puncta transeuntem. Itaque sectio ejus longitudinem habet, cujus autem sectio habet longitudinem, id ipsum habet latitudinem.

Datis quatuor punctis in idem planum non cadentibus resultat profundum, seu id in quo sumi potest aliquid quod Terminus non est, seu quod non potest ei esse commune cum altero nisi pro parte in ipsum immerso.

(6) *Fourth,* it follows that a plane relates in the same way on both sides, and thus has neither concavity nor convexity.

(7) And so to a point relating in whatever way to A, B, C together, or to the plane determined by them, another corresponding point can be given relating in the same way to these three points taken together, since there is no reason for a distinction.

(8) Now a plane is endowed with width, since it can be sectioned by a straight line passing through a given pair of its points. And so a section of it has length; and that whose section has length, itself has width.

Given four points not falling on the same plane, there results depth, or that in which something can be taken that is not a boundary, i.e. which cannot be common to it and some other that isn't a part immersed in it.

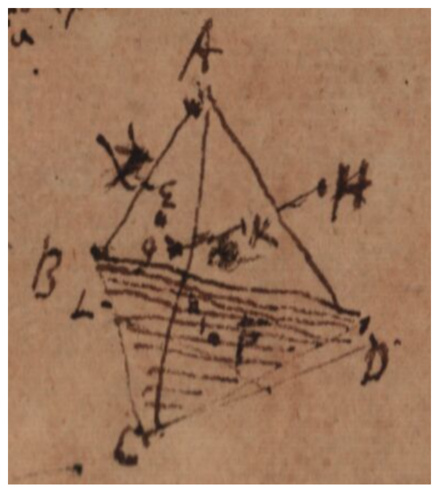

Figure 1

Sunto quatuor puncta A, B, C, D (fig. 1). Hinc dantur sex rectae AB, AC, AD, BC, BD, CD; sed sufficiunt AB, AC, AD, nam tres reliquae ex his nascuntur. Prosultantibus his tribus rectis, prosultant et omnia earum puncta, et rectae conjungentes duo quaevis diversarum rectarum puncta, locusque adeo harum rectarum omnium. Porro habemus et quatuor plana per A, B, C, per A, B, D, per A, C, D, per B, C, D, quorum duo quaevis sectionem habent rectam communem; verb. gr. plana per A, B, C et per B, C, D habent communem sectionem rectam per B, C. Haec quatuor plana claudent spatium quatuor Triangulis planis ABC, ABD, ACD, BCD, quae spatii ambitum facient. Et recta quaevis EF, duo puncta E et F duorum planorum ABC, BCD conjungens, habet omnia puncta ut G intra hoc spatium, ita ut ex puncto G rectae extremis interjecto nulla recta educi possit, quae non in ambitum cadat. *Claudunt* autem *spatium* quae ambitum constituunt plenum, nempe talem, ut linea quaecunque ducta in parte ambitus, ubi pervenit ad ejus partis extremum, continuari possit in alia ambitus parte. Ex. gr. recta AL in parte Ambitus ABC ducta, ubi pervenit ad extremum ejus L, continuari non posset in alia ambitus parte, si abesset triangulum BCD, quod cum caeteris tribus ABC, ABD, ACD spatii claudendi opus absolvit. Ponatur illa recta produci quantum opus est ad punctum H, quod magis absit a puncto G, quam omnia puncta triangulorum ABC, ABD, ACD, BCD. Ex puncto H agantur normales in quatuor plana et itidem ex puncto G, reperietur aliquod ex his planis planum, respectu cujus normales amborum non sint ad easdem partes; aliquod ex his planis debet GK secare in triangulo cujus planum est continuatio; dico rectam GH si opus productam cadere in aliquod ex his planis. Sumatur aliud quodcunque punctum H, ajo rectam GH productam si opus occurrere uni triangulorum quatuor ut ipsi ABD in K, planum per E, F, H secabit plana ABD, ACD, BCD, quae tres rectae constituent triangulum cujus duo latera cadent in duo

Let there be four points A, B, C, D. [See Figure 1.] From this are given six lines AB, AC, AD, BC, BD, CD; but AB, AC, AD suffice, since the remaining three arise from these. With these three lines prosulting, all of their points also prosult, as well as the lines conjoining any two points of distinct lines, and so also the locus of all these lines. Furthermore we have also four planes, through $A.B.C$, through $A.B.D$, through $A.C.D$, through $B.C.D$, of which any two have a common line as a section; for example the planes through $A.B.C$ and $B.C.D$ have as a common section the line through $B.C$. These four planes enclose a space with four planar triangles ABC, ABD, ACD, BCD, which form the ambit of the space. And each line EF connecting two points E and F of the two planes ABC, BCD has all points G inside this space, so that from a point G, interposed between the extremes of the line, no line can be drawn out that does not fall upon the ambit. Now[11] those *enclose space* which constitute a full ambit, namely one such that any curve drawn in a part of the ambit, when it reaches an extreme of that part, can be continued in another part of the ambit. For example, the line AL drawn on the part ABC of the ambit, when it reaches its extreme L, could not be continued in another part of the ambit if the triangle BCD were missing, which together with the other three triangles ABC, ABD, ACD completes the task of enclosing space. Take any other point H; I say that the line GH, extended if necessary, meets one of the four triangles, such as ABD, at K; the plane through E, F, H will intersect the planes ABD, ACD, BCD; these three lines constitute a triangle, of which two sides will fall on two of the three aforementioned triangles, and the point G will fall inside this triangle. Therefore any line passing through G, and therefore the line GH, will intersect one of these two sides; therefore the line GH meets one of the triangles ABD, ACD, BCD at K. In the same way it will be shown to meet one of the other three triangles at the other extreme as well. But we will show the matter more briefly. Suppose that line is extended as far as necessary to a point H which is farther from the point G than all points of the triangles ABC, ABD, ACD, BCD. From the point H let normals be directed to the four planes, and likewise from the

[11] These sentences are inserted in the right margin of the manuscript.

ex triangulis tribus dictis, et punctum G intra hoc triangulum cadet. Recta ergo quaecunque transiens per G alterutrum ex illis duobus lateribus secabit, ergo et recta GH, ergo recta GH occurrit uni ex triangulis ABD, ACD, BCD in K. Eodem modo ostendetur, et alio extremo occurrere uni ex aliis tribus triangulis. Sed brevius rem ostendemus.

point G; from these planes some plane will be found with respect to which the normals of both are not on the same side; some one of these planes must intersect GH[12] within a triangle of which the plane is a continuation.[13]

[12]In the manuscript, it appears that Leibniz originally had GH and then wrote GK over top of it. The correction seems to have been a mistake.

[13]This paragraph is an alternative argument to what Leibniz gave in the previous paragraph. The manuscript here shows significant editorial work. Leibniz has crossed out portions of the previous paragraph, circled the previous paragraph, and added this alternative argument together with the connecting clause 'But we will show the matter more briefly'. In the previous paragraph, Leibniz inserted what appears to be an alternative version of the first sentence, namely 'I claim that the line GH, extended if necessary, falls on one of these planes', which we conjecture he added upon realizing his new argument only shows that GH would intersect one of the *planes*, and not necessarily fall in one of the triangles.