G. W. Leibniz

De Quantitate

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Initia Mathematica

De quantitate

Determinantia sunt quae simul non nisi uni soli competunt. Ut duo extrema A. B. non nisi uni competunt rectae.

Initia Mathematica

De Quantitate

Determiners are things that are simultaneously compatible with only a single one. As two extremes A, B are compatible with just one straight line.

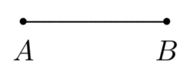

Figure 1

Coincidentia sunt; quae plane eadem sunt, tantumque denominatione differunt, ut via ab A ad B. a via a B ad A.

Coincidents are things that are completely the same and differ only by denomination, such as the path from A to B and the path from B to A.

Congrua sunt, quae si diversa sunt, non nisi respectu ad externa discerni possunt, ut quadrata C et D. Nempe quod eodem tempore sunt in diverso loco vel situ; vel quod unum C est in materia aurea, alterum D in argentea. Ita congruunt libra auri et libra plumbi. Dies hodiernus et hesternus. Punctum quodlibet congruit cuilibet alteri; ut et instans instanti.

Congruents are things that, if they are distinct, cannot be distinguished except with respect to something external, such as the squares C and D, since of course they are in distinct places or situations at the same time, or because one C is in a gold material, the other D in silver. Likewise a gold pound and a silver pound are congruent, and the days today and yesterday. Any point is congruent to any other, as also any instant to any instant.

Aequalia sunt quae vel congruunt (exempli gratia triangula

Equals are things that are either congruent (for example, trian-

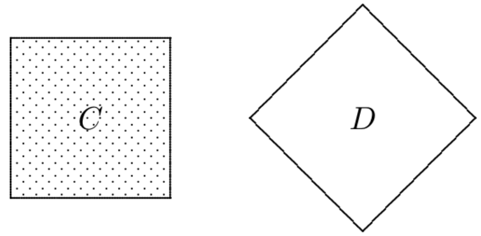

Figure 2

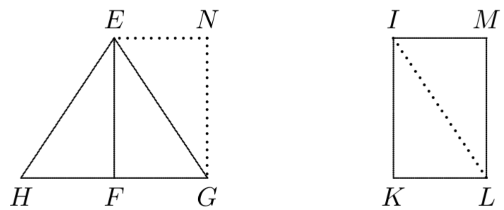

Figure 3

EFH. *EFG*. *IKL*. *LMI*. *GNE* item rectangula *EFGN* et *IKLM*) vel per transformationem congrua reddi possunt. (Ut triangulum *HEG* rectangulo *IKLM*. quia parte ipsius *HEG* nempe *EFH* transposita in *GNE*, quod fieri potest quia congruunt, tunc *HEG* transformatum erit in *FGEN* congruum ipsi *IKLM*. Itaque *HEG* et *IKLM* aequalia dicentur.) Itaque definiri possunt *Aequalia,* quae resolvi possunt in suas partes diversas singula singulis alterius congruentes.

Similia sunt in quibus per se singulatim consideratis inveniri non potest quo discernantur ut duo sphaerae vel circuli (vel duo cubi aut duo quadrata perfecta) *A*. et *B*. Ut si solus oculus sine aliis membris fingatur nunc esse intra sphaeram *A* nunc

gles *EFH*, *EFG*, *IKL*, *LMI*, *GNE*, and likewise rectangles *EFGN* et *IKLM*), or can be rendered congruent by a transformation (as triangle *HEG* to rectangle *IKLM*, since if part of *HEG*, namely *EFH*, is transposed to *GNE*, which can be done because they are congruent, then *HEG* is transformed to *FGEN*, congruent to *IKLM*; and so *HEG* and *IKLM* are called equals). Therefore *equals* can be defined as things that can be resolved into separate parts, each one congruent individually to one part of the other.

Similars are things in which nothing may be discovered by which they can be distinguished, if they are considered in themselves individually, such as two spheres or circles (or two cubes or two perfect squares) *A* and *B*. If the eye only, without any other member, is imagined to be now

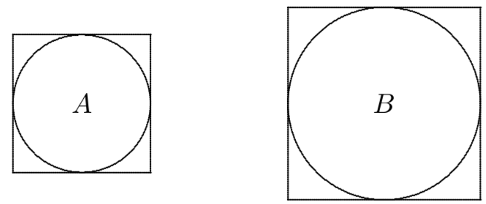

Figure 4

intra sphaeram B. non poterit eas discernere; sed poterit si ambas simul spectet, vel si secum membra alia corporis aliamve mensuram introrsum afferat, quam nunc uni nunc alteri applicet. Itaque ad similia discernenda opus est vel compraesentia eorum inter se, vel tertii cum singulis successive. [At in dissimilibus aliqua partium proportio notata in uno, quae non notatur in altero sufficit ad discernendum sigillatim. De quo postea pluribus.]

inside sphere A, now inside sphere B, it will not be able to distinguish them; it can, though, if both are seen simultaneously, or if another limb of the body or another measure is brought inside with it, which is applied now to one, now to the other. Thus, to distinguish similars we either need the co-presence of them with each other, or of a third thing with each in succession. [But in dissimilars, something noted in one by the proportion of its parts, which is not noted in the other, suffices for distinguishing them individually. More on this later.][1]

Figure 5

Homogenea sunt, quae aut similia sunt aut similia transformatione reddi possunt. Duae rectae sunt homogeneae, quia similes; sed et recta et arcus circuli homogeneae res sunt, quia circulus in rectam extendi potest.

Homogeneous things are those which either are similar, or can be rendered similar by a transformation. Two lines are homogeneous since they are similar, but also a line and a circular arc are homogeneous since the circle can be stretched into a line.

Si plura sint ut A vel B. et unum ut C. sintque in aliquo convenientia, et in his homogeneum insit ipsis A et B commune, omnia vero in ipsis homogenea, sint ipsi C communia;

Suppose there is a plurality, say A and B, and a unity, say C, and they fit together in something, and nothing homogeneous is in A and B in common, but all homogeneous things in them are common to C; then

[1] Brackets original.

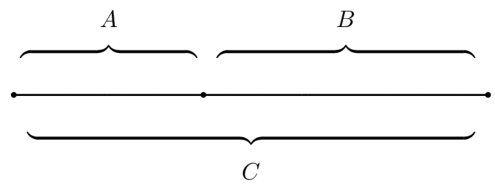

Figure 6

tunc illa plura dicentur *partes integrantes* unum illud dicetur *totum*.

Homogenea etiam definire possumus, quae in aliquo conveniunt, et in quo conveniunt alia quae in uno quoque eorum indefinite assumi possunt.

Minus est quod alterius *(Majoris)* parti aequale est. *Quantitas* est id quod rei competit, quatenus habet omnes suas partes, sive ob quod alteri (homogeneae cuicunque) aequalis major aut minor dici, sive comparari potest.[3]

the plurality are called *integrating parts* and the unity is called the *whole*.[2]

We could also define *homogeneous* things as those which fit together in something, and in which others also fit together that can be assumed in either one of them indefinitely.

The *lesser* is that which is equal to a part of the other the (*greater*). *Quantity* is that which belongs to a thing insofar as it has all its parts, or on account of which it may be said to be equal to, greater than, or less than another thing (any homogeneous thing), i.e. it may be compared.

[2]The manuscript has a variety of deletions and insertions in this sentence.
[3]Editorial note (print p. 296, l. 8): *Dazu, am unteren Rand:* insere huc schedam adjectam aliunde abscissam sub signo ⊕

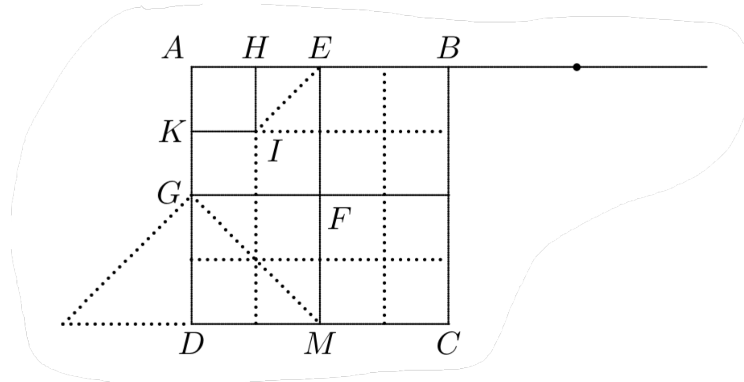

Figure 7

Quantitas rei, ex. gr. fig. 1. areae *ABCD* exprimitur per numerum ex. gr. quaternarium posito aliam rem, ut pedem quadratum *AEFG* sumtum esse pro Mensura primaria seu Unitate reali. Est enim *ABCD* quatuor pedum quadratorum. Si vero alia assumeretur unitas *AHIK* quae est quadratum semipedis *AH*, quantitas areae *ABCD* esset 16. Itaque pro eadem quantitate diversus proveniet numerus prout assumitur unitas. Et proinde quantitas non est numerus definitus, sed materiale numeri seu numerus indefinitus, assumta certa mensura definiendus. Quantitates ergo exprimuntur vel numeris definitis, ut 1. 2. vel indefinitis seu literis aliisve characteribus. $a.b.$ ⊙ . ☽.[4]

Numerus est homogeneum Unitatis. Adeoque comparari cum unitate eique addi adimique potest. Estque vel aggregatum unitatum qui dicitur *integer*, ut 2. (seu 1 + 1.), item 3. 4. (seu 2 + 1 sive 1 + 1 + 1) vel aggregatum partium aliquotarum unitatis, qui dicitur *Fractus*, ut si unitas ex. gr. pes *AB* sit divisus in quatuor partes quartas, tunc res ut linea *BH* quae habet tres quartas pedis seu ter $\frac{1}{4}$ ita exprimetur $\frac{3}{4}$ isque

The *quantity* of a thing, for example of the area *ABCD* in Figure 7, is expressed by a number, for example four, with another thing such as the square foot *AEFG* being taken for a primary measure or real unit. Indeed, *ABCD* is four square feet. But if another unit *AHIK* is assumed, which is the square of the half foot *AH*, the quantity of the area *ABCD* will be 16. And so various numbers will result for the same quantity according to the unit assumed. And hence a quantity is not a definite number, but the material of a number, or an indefinite number, to be defined when a fixed measure is assumed. Quantities, therefore, are expressed by definite numbers, like 1, 2, or indefinite, by letters or other characters a, b, ⊙, ☽.

A *number* is something homogeneous with unity and thus can be compared with unity and added to and subtracted from it. It is either an aggregate of unities, which is called an *integer*, such as 2 (or 1 + 1), and likewise 3, 4 (or 2 + 1 or 1 + 1 + 1), or else an aggregate of some parts of unity, which is called a *fraction*; if a unity, for example a foot *AB*, is divided in four quarter parts, then something such as the line *BH*, which has three quarters of a foot, or three $\frac{1}{4}$, will thus be

[4]Editorial note (print p. 297, l. 2): *Dazu am Rand Einfügungszeichen:* ⊕

fractus interdum reduci potest ad integrum, ut AB sive 2 est quater AH seu 4 dimidiae sive $\frac{4}{2}$; vel denique numerus est alio quodam modo per relationem ad unitatem determinatus; quae quidem relationes possunt esse infinitae sed maxime solennes sunt per radices. Nempe sit numerus 4 (pro quadrato $ABCD$ fig. 1.) quaeritur ejus radix quadrata (seu latus AB.) id est numerus qui per se multiplicatus facit 4. Is numerus erit 2. Itaque cum 2.2 sive aa sit 4. $\sqrt{4}$ ($\sqrt{aa}$) est 2 (a). Atque hoc casu radix reduci potest ad numerum communem seu *rationalem*. Sed interdum haec reductio non succedit. Ex. gr. quaeritur numerus qui per se ipsum multiplicatus faciat 2. Is neque est integer, (alioqui enim cum necessario sit minor quam 2 foret, unitas, at unitas per se multiplicata facit 1.) neque est fractus, quia omnis fractus per se ipsum multiplicatus producit alium fractum, ut $\frac{3}{2}$ producit $\frac{9}{4}$ sive $2+\frac{1}{4}$. Itaque non est numerus nisi irrationalis ut vocant, sive potius ineffabilis, ἄλογος *surdus*, qui sic scribitur $\sqrt{2}$ vel $\sqrt{q}$ 2 vel $\sqrt[2]{2}$ id est radix quadratica de 2. Posito enim hunc numerum esse, y: tunc ejus quadratum yy sive y^2 erit 2. Et ut appareat hunc numerum esse in rerum natura, *in fig.* 2. ducatur AF diagonalis quadrati $AEFG$. Sit AG 1. nempe unus pes, cujus quadratum est 1 (nempe ipsum spatium $AEFG$ seu unus pes quadratus,) tunc ipsa AF quam vocabimus y erit $\sqrt{2}$. Nam ejus quadratum yy. $AFBM$ est 2, (nempe duplus pes quadratus). Est enim quadratum $AFBM$ duplum quadrati $AEFG$. Nam dimidium ejus triangulum AFB. toti $AEFG$ aequale est. Cum ergo numerum definierimus homogeneum unitati, utique debet aliquis esse numerus cujus ea sit relatio ad unitatem, quae est rectae AF ad rectam AG, sive posito AG esse 1. debet esse numerus quo exprimatur quantitas ipsius AF qui dicetur esse $\sqrt{2}$. AB autem erit 2.

expressed $\frac{3}{4}$; and sometimes a fraction may be reduced to an integer, as AB or 2 is four of AH, or 4 halves, or $\frac{4}{2}$; or, finally, a number is determined in some other way by relation to unity, and in fact these relations could be infinite, but the most customary are by radicals. Indeed, for the number 4 (for the square $ABCD$, Figure 7), suppose its square root is sought (or the side AB); it is the number which, multiplied by itself, makes 4; the number is 2, and so since $2 \cdot 2$ or aa is 4, $\sqrt{4}$ ($\sqrt{aa}$) is 2 (a). And in this case the radical can be reduced to a common number or a *rational*. But sometimes this reduction does not succeed. For example, if a number is sought which multiplied by itself would make 2, it certainly is not an integer (for otherwise, since it is necessarily less than 2, it would be unity, but unity multiplied by itself makes 1), and neither is it a fraction, since every fraction multiplied by itself produces another fraction, as $\frac{3}{2}$ produces $\frac{9}{4}$ or $2+\frac{1}{4}$; and so the number is nothing but *irrational*, as they say, or rather ineffable, *alogos*, *surd*, which is written another way as $\sqrt{2}$, or $\sqrt{q}$ 2, or $\sqrt[2]{2}$, that is, the quadratic root of 2, for with this number set as y, then its square yy or y^2 will be 2. And so that it is clear that this number is in the nature of things, in Figure 8 let the diagonal AF be drawn in the square $AEFG$. Let AG be 1, namely one foot, whose square is 1 (namely the space $AEFG$, or one square foot), then AF, which we call y, will be $\sqrt{2}$; for its square yy, $AFBM$, is 2 (namely double a square foot); the square $AFBM$ is in fact double the square $AEFG$, for its half, triangle AFB, is equal to the whole $AEFG$. Since, therefore, we define number as something homogeneous to unity, certainly there should be some number having the ratio to unity that the line AF has to the line AG, or with AG set as 1, the number expressed by the quantity of AF should be what is called $\sqrt{2}$; AB, moreover, will be 2.

Itaque si sit *Scala AB*. fig. 2 divisa in partes aequales duas, quatuor, octo, sedecim etc. atque ita porro subdivisa quantum libuerit, huic scalae utique recta quaelibet ipsa scala minor

Therefore if a *scale AB* (Figure 8) is divided into two equal parts, and four, eight, sixteen, etc., and further subdivided as much as one pleases, then certainly any line smaller than the scale can be placed

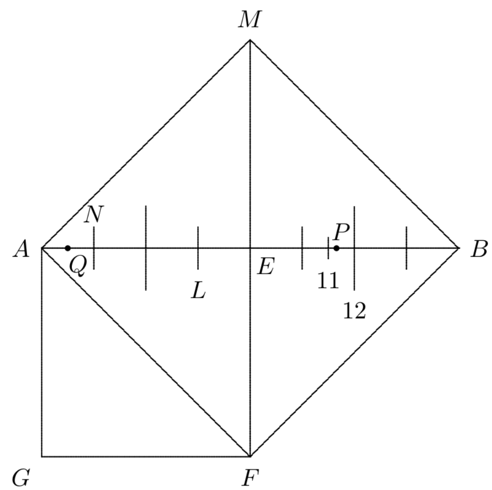

Figure 8

applicari, adeoque numeris explicari potest, et quidem vel exacte vel propemodum *Exacte* quidem, quando scilicet incipiens ab initio scalae A, incidit in aliquod punctum divisionis L ut AL cujus proinde numerus in partibus scalae habetur ex. gr. AE existente 1. tunc AL erit $\frac{3}{4}$ et tunc recta AL est scalae commensurabilis; id est datur earum mensura communis seu recta AN $\left(\frac{1}{4}\right)$ quae repetita tam scalam AB quam rectam AL efficit seu *metitur*. *Propemodum* vero numeris recta scalae applicata explicari potest, quando non incidit in punctum divisionis, quantum vis scala subdividatur et quocunque modo instituatur divisio. Et talis recta ut AF est cum scala incommensurabilis, adeoque numeris rationalibus sine effabilibus explicari nequit, nisi propemodum. Quoniam tamen necesse est, ut AF scalae applicata seu translata in AP. incidat saltem inter duo divisionis puncta uti certe incidit inter 11 et 12, posito scalam AB in sedecim partes aequales esse divisam, quarum quaelibet est pars octava unitatis vel

on the scale and thus expressed explicitly by numbers [*explicari numeris*], either exactly or approximately. *Exactly* when, beginning at the start of the scale A, it falls on some point of division L, as AL, whose number is thus obtained in the parts of the scale. For example, with AE being 1, then AL is $\frac{3}{4}$, and then the line AL is commensurable with the scale, that is, there is a common measure for them, i.e. the line AN $(\frac{1}{4})$, which produces [*efficit*] by repetition, or *measures*, the scale AB as well as the line AL. On the other hand, a line applied to a scale can be *approximately* expressed explicitly by numbers when it does not fall on a point of division however much the scale is subdivided and in whatever way the division is established. And a line such as AF is incommensurable with the scale to the extent that it cannot be expressed explicitly by a rational or effable number except approximately. Nevertheless it is necessary that AF placed on the scale, i.e. translated to AP, at least falls between two division points, as of course here it falls between 11 and 12, supposing the scale AB to be divided into sixteen equal parts any of which is the eighth part of pedis AE. Hinc si AG vel AE latus quadrati sit pes, tunc diagonalis AF vel AP incidet inter $\frac{12}{8}$ (sive $\frac{3}{2}$) et $\frac{11}{8}$ pedis, adeoque si ⟨ipsi AF sive $\sqrt{2}$ vel y attribuas $\frac{3}{2}$, nimium, si $\frac{11}{8}$, parum tribues (nam quadratum a $\frac{3}{2}$ est $\frac{9}{4}$ id est plus quam $\frac{8}{4}$ seu 2 seu yy, et quadratum ab $\frac{11}{8}$ est $\frac{121}{64}$ quod est minus quam $\frac{128}{64}$ seu 2), error tamen semper⟩ minor una parte minima, in quam hoc loco unitas in scala divisa est, id est minor quam $\frac{1}{8}$. Et $\frac{1}{8}$ sive AQ erit propemodum mensura communis unitatis sive etiam scalae, et ipsius AF. Et quanto magis subdivisa erit scala eo minor erit error adeoque erit tam parvus quam quis velit, sive minor reddi potest quovis errore assignabili. Itaque AF et AG etsi sint incommensurabiles, sunt tamen *homogeneae,* seu comparabiles, et inveniri potest mensura communis tam prope exacta ut error seu residuum sit minus data quantitate. Atque hoc est fundamentum *appropinquationum,* et *computandarum Tabularum,* itemque Logisticae binariae vel sexagenariae vel etiam *decimalis,* si quidem scala in decem partes dividatur, et harum quaelibet in alias decem subdividatur, idque quantumlibet continuetur. Etsi enim scalae instrumentis satis subdividi non semper possint, mente tamen sive calculo ad summam exactitudinem, quae quidem *in praxi* optari possit, procedi potest. Quod quomodo fiat infra apparebit.

the unit or the foot AE.[5] Hence if AG or AE, the side of a square, is a foot, then the diagonal AF or AP falls between $\frac{12}{8}$ (or $\frac{3}{2}$) and $\frac{11}{8}$ of a foot, and thus if you allot $\frac{3}{2}$ to AF, i.e. $\sqrt{2}$ or y, it is too much, and if $\frac{11}{8}$, too little is allotted (for the square of $\frac{3}{2}$ is $\frac{9}{4}$, that is, more than $\frac{8}{4}$ or 2 or yy, and the square of $\frac{11}{8}$ is $\frac{121}{64}$, which is less than $\frac{128}{64}$ or 2); nonetheless the error is always smaller than a smallest part into which unity in the scale is divided here, that is, less than $\frac{1}{8}$. And $\frac{1}{8}$ or AQ will be approximately a common measure for the unity (or indeed the scale) and AF. The more a scale is subdivided, the smaller the error will be, and so it will be as small as anyone wants, i.e. it can be rendered smaller than any assignable error. Therefore even if AF and AG are incommensurable, they are nevertheless *homogeneous* or comparable, and one can discover a common measure sufficiently precisely that the error or remainder is smaller than a given quantity. And in fact this is the foundation of *approximations*, and *tabular computations*, and likewise the binary or sexagesimal or even *decimal* logistic, when indeed a scale is divided into ten parts, and any of them subdivided into another ten, continuing as far as desired. In fact, even if one cannot always divide the scale sufficiently with instruments, nevertheless, with the mind or by calculation, one can at least proceed to the highest precision that could be desired *in practice*. How this is done will appear later.

[*Verworfener Abschnitt*]

Ratio est homogeneum aequalitatis, adeoque si aequalitas spectetur ut unitas, quia tunc consequens in antecedente non nisi semel continetur, ratio erit numerus qui oritur ex divisione antecedentis per consequens, vel numerus quo exprimeretur antecedens, si consequens esset mensura primaria per quam caetera exprimi deberent sive Unitas. Etsi fortasse alia nunc unitas seu mensura primaria assumta sit. Ita ratio

[The next portion was cancelled.]

A *ratio* is something homogeneous with equality, and so if equality is viewed as the unit, since then the consequent will be contained in the antecedent only once, the ratio will be the number which arises from the division of the antecedent by the consequent, or the number by which the antecedent would be expressed if the consequent were the primary measure by which others should be expressed, or the unit. Even if now perhaps another primary measure or unit is adopted.

[5] Leibniz edited this sentence heavily. We have ignored a leading *'quoniam'* that it appears he forgot to cancel.

trium pedum ad pedem est tripla rationis quam habet pes unus ad seipsum, seu tripla unitatis; id est ternarius. Nam et si pes sit 1. tres pedes erunt 3. Unitas autem est ratio rei ad seipsam vel ad aequalem, cum una quantitas in altera non nisi semel contineatur. Sed Ratio unius pedis ad tres, id est ad unum tripodem (sumtis tribus pedibus pro unitate) est subtripla, seu tertia pars rationis quam habet tripus ad seipsum, seu tertia pars unitatis. Nam et si tripus esset 1. pes foret $\frac{1}{3}$ scilicet unitatis nempe tripodis. Ratio pedis ad dipodem est subdupla seu dimidia rationis quam habet dipus ad seipsum, id est unitatis. Ergo est $\frac{1}{2}$. Nam et si dipus esset 1. pes foret $\frac{1}{2}$ scilicet unitatis nempe dipodis. Et ratio trium pedum ad dipodem seu 3 ad 2 est ter ratio unius pedis ad dipodem seu tripla subdupla, sive tres dimidiae sive $\frac{3}{2}$. Adeoque si dipus esset 1. tunc tres pedes forent tres dimidiae seu $\frac{3}{2}$ unius dipodis.

Proportionalia sunt quorum eadem est ratio. Ut numeri 3 ad 2 eadem est quae numeri 6 ad 4. Est enim rectae AC ad rectam BC eadem semper ratio, adeoque et numerorum quibus AC et BC exprimuntur eadem erit ratio licet diversa assumta sit unitas.

Thus the ratio of three feet to a foot is triple the ratio a foot has to itself, or triple the unit, that is, three. For if a foot is 1, three feet will be 3. Unity, moreover, is the ratio of a thing to itself, or to an equal, when the one quantity is contained in the other only once. But the ratio of one foot to three, that is, to one triple-foot [*tripode*] (three feet being taken for the unit) is sub-triple, i.e. the third part of the ratio which the triple-foot has to itself, or the third part of the unit. Indeed if the triple-foot is 1, the foot would of course be $\frac{1}{3}$ of the unit, namely of the triple-foot. The ratio of the foot to the double-foot [*dipode*] is sub-double, or half of the ratio which the double-foot has to itself, that is, [half] of the unit. It is therefore $\frac{1}{2}$. For instance, if the double-foot is 1, the foot would of course be $\frac{1}{2}$ of the unit, namely the double-foot. And the ratio of three feet to the double-foot, or 3 to 2, is three times the ratio of one foot to the double-foot, or triple the sub-double, or three of the half, or $\frac{3}{2}$. And thus if the double-foot is 1, then three feet would be three of the half, or $\frac{3}{2}$, of one double-foot.

Things are *proportional* whose ratio is the same. As [the ratio] of the number 3 to 2 is the same as that of the number 6 to 4. For the ratio of the line AC to the line AB is always the same, and therefore the ratio of the numbers by which AC and AB are expressed will also be the same even if a different unit is assumed.[6]

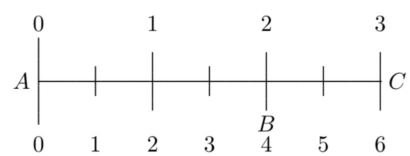

Figure 8a

[*Fortsetzung des gültigen Textes*]

[Cancelled portion ends here.]

[6] Leibniz wrote BC instead of AB in this sentence.

Dimensiones sunt quantitates diversae (interdum heterogeneae[)], *quae in se invicem duci* intelliguntur, ita scilicet ut una tota applicetur cuilibet parti alterius. Exempli causa ex ductu latitudinis AB duorum pollicum in longitudinem BC trium pollicum linearium (ita ut anguli A. B. C. e. f semper congruant seu iidem sint, qui dicuntur recti, qui est simplicissimus lineam rectam in rectam ducendi modus) fit rectangulum ABC. quod est duarum dimensionum, et sex pollicum sed quadratorum.

Dimensions are distinct (sometimes heterogeneous) quantities that are understood to be compounded by each other,[7] so that the whole of one is applied [*applicetur*] to each part of the other. For example, by drawing a latitude AB of two inches to a longitude BC of three linear inches (so that the angles A, B, C, e, f are always congruent, i.e. are the same, what are called right, which is the simplest way of drawing a straight line to a straight line) the rectangle ABC is formed, which has two dimensions, and is six inches—but square inches.

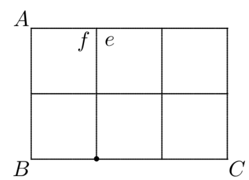

Figure 9

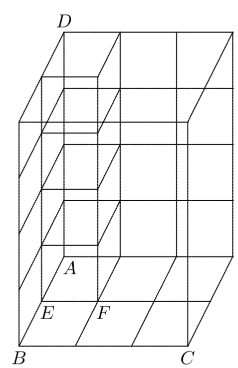

Figure 10

Ex ductu longitudinis CB, 2. latitudinis BA, 3 altitudinis

By drawing a longitude CB, 2, latitude BA, 3, and altitude AD, 4,

[7]Latin *in se invicem duci*, literally ‘drawn to themselves mutually’.

AD, 4 in se invicem fit rectangulum solidum *CBAD* quod est trium dimensionum, seu viginti quatuor pollicum cubicorum (2 $\wedge$ in 3 $\wedge$ in 4). Cuilibet enim ex baseos *ABC* sex quadratillis seu pollicibus quadratis, (ut quadratillo *AEF*) insistunt quatuor cubuli seu unitates cubicae sive pollices cubici (nempe columna seu prisma *FEAD* ex quatuor pollicibus cubicis sibi impositis constans[)]. Nec vero putandum ut hactenus crediderunt, dimensionem spectari in solis figuris, adeoque rem altioris gradus seu plurium dimensionum quam trium dimensionum esse imaginariam. Etsi enim spatium per se habeat tantum tres dimensiones, corpus tamen potest habere multo plures, ex. g. duo corpora unum aureum alterum argenteum habent praeter considerationem molis seu spatii, quod occupant, etiam considerationem gravitatis specificae, quae in qualibet parte molis spectatur. Ita gravitate specifica pollici cubici argenti posita ut 55 auri ut 99 unciarum (ea enim fere proportio est) erit pondus solidi *CBAD* si aureum sit 2 $\wedge$ 3 $\wedge$ 4 $\wedge$ 99 (seu 24 $\wedge$ 99 sive) 2376 unciarum; sin argenteum sit solidum erit unciarum 2 $\wedge$ 3 $\wedge$ 4 $\wedge$ 55 seu (24 $\wedge$ 55) 1320 unciarum. Itaque pondera ista sunt quatuor dimensionum; ex ductu scilicet molis seu spatii tridimensi, in corpus ipsum seu pondus. Potest etiam praeter pondus mortuum accedere impetus ex descensu gravis aliquamdiu continuatus; unde nascitur percussio quae est quinque dimensionum, ex mole tridimensa, corporis ponderositate et tempore lapsus in se invicem ductis. Ita si una ulna quadrata panni valeat tres nummos imperiales duae ulnae valebunt bis tres imperiales seu sex. Et pretium hoc est duarum dimensionum, quod si idem pannus sit quatuor ulnas latus, erit pretium ejus 2.3.4 seu 24 imperialium, adeoque trium dimensionum ex ductu in se invicem longitudinis, latitudinis, pretiositatis id est pretiositatis seu bonitatis intrinsecae in quantitatem seu bonitatem extrinsecam. Ita pretium aggeris est quatuor dimensionum, spectatur enim in eo longitudo quae sit pedum 100. latitudo 12. altitudo 20, et firmitas seu bonitas intrinseca sit talis, ut pes cubicus valeat decem num-

to each other, a solid rectangle *CBAD* is formed (Figure 10), which has three dimensions, or twenty four cubic inches (2 $\overset{\frown}{\text{by}}$ 3 $\overset{\frown}{\text{by}}$ 4); indeed on any of the six little squares or square inches of the base *ABC* (like the little square *AEF*) there stand four cubes, or cubic units, or cubic inches (namely a column or prism *FEAD* consisting of four cubic inches placed on each other). Nor should one think, as has been believed up to now, that dimension is observed in figures alone, and thus that something of higher degree, or of more dimensions than three dimensions, is imaginary. Even though space *per se* has only three dimensions, a body can nevertheless have many more, for example two bodies, one of gold, another of silver, have, beyond the consideration of the bulk or space that they occupy, yet another consideration of specific gravity, which is observed in any part of the bulk. Thus, setting the specific gravity of a cubic inch of silver at 55, and of gold at 99 ounces (that is nearly the proportion), the weight of the solid *CBAD*, if it is gold, will be $2 \frown 3 \frown 4 \frown 99$ (or $24 \frown 99$) or 2376 ounces; but if the solid is silver, it will be $2 \frown 3 \frown 4 \frown 55$ ounces or ($24 \frown 55$) 1320 ounces. Therefore those weights have four dimensions, namely by compounding the three-dimensional bulk or space with the body itself, or the weight. Besides the dead weight, one can also add an impetus from the descent of a heavy body continued over some time; from this arises an impact that has five dimensions: from the three-dimensional bulk, the weight of the body, and the elapsed time, compounded with each other. In this way if one square arm of cloth is worth three imperial coins, two arms will be worth twice three imperials, or six. And this price has two dimensions, for if the same cloth is four arms on the side, its price will be $2 \cdot 3 \cdot 4$ or 24 imperials, and so for three dimensions by compounding the longitude, latitude, and price with each other, that is, the price, or intrinsic value, with the quantity, or extrinsic value. In this way the price of a heap has four dimensions, for one sees in it a longitude of 100 feet, a latitude of 12, altitude of 20, and firmness or intrinsic value such that a cubic foot is worth ten coins. Then its value will be $100 \frown 12 \frown 20 \frown 10$ coins or 240000, whence compounding a dimension by a dimension is a real exhibit of a mental multiplication.

mos. Erit valor ejus 100. $\wedge$ 12. $\wedge$ 20. $\wedge$ 10. nummorum seu 24000[0] ut proinde ductus dimensionis in dimensionem, sit exhibitio realis, multiplicationis mentalis.

Ex his definitionibus sequentia Axiomata duci possunt.

From these definitions the following axioms can be derived.

Quae iisdem (vel coincidentibus) determinantur (eodem scilicet modo) coincidunt. Ut coincidunt duae rectae, quarum duo extrema coincidunt.

Things which are determined (that is, in the same manner) by the same things (or by coincidents) are coincident. As two lines coincide whose two extremes coincide.

Quae coincidunt ea multo magis congruunt. Seu idem congruit sibi.

Things which coincide are even more so congruent. I.e. the same thing is congruent to itself.

Quae congruentibus determinantur (eodem scilicet modo) congrua sunt. Ut quia triangulum datur, datis tribus lateribus, hinc si tria trianguli latera respondentia respondentibus congruant congruent triangula.

Things which are determined by congruents (that is, in the same manner) are congruent. So, since a triangle is given when its three sides are given, then if the three sides of a triangle are respectively congruent to corresponding ones, the triangles will be congruent.

Quae congruunt ea multo magis aequalia sunt.

Things which are congruent are even more so equal.

Aequalia eadem sumta mensura eodem numero exprimuntur; sive ejusdem sunt quantitatis, cum enim inter se congrua reddi possint, eidem mensurae primariae seu unitati eodem modo repetitae, eodem modo congruere poterunt. Unde idem prodit numerus.

Equals are expressed by the same number with the same measure being taken, i.e. they are of the same quantity; indeed since they can be rendered congruent to each other, they can be made congruent in the same way to the same primary measure, or unity, repeated in the same way. The same number results from this.

Aequalia eodem modo secundum quantitatem tractata exhibent aequalia.

Equals treated in the same way according to quantity yield [*exhibent*] equals.

Similia similiter tractata exhibent similia, ideo Quae similiter similibus determinantur similia sunt. Determinari autem intelligo iis conditionibus designari quae simul non nisi in unum cadere possunt: Itaque quod ita determinatur, id plane exhibetur.

Similars treated similarly yield similars, and for that reason things determined similarly by similars are similar. By ‘determined’ I mean things that are designated by conditions which can hold simultaneously in only one thing. And so what is thus determined is clearly yielded.

Similia et aequalia simul, sunt congrua. Nihil enim superest quo discerni possint, sive sigillatim sive simul spectentur nisi referantur ad externa; ut locum et tempus aliaque accidentia.

Things simultaneously similar and equal are congruent. Indeed nothing remains by which they could be distinguished, whether individually or observed simultaneously, unless they are related to something external, such as place and time and other accidents.

Ratio non est nisi inter homogenea. Patet ex definitione.

There is no ratio except among homogeneous things; that is clear from the definition.

Quorum unum altero majus minus aut aequale est homogenea sunt. De aequalibus manifestum est, possunt enim congrua reddi, adeoque et similia. Minus quoque majori homogeneum, quia ejus parti aequale adeoque homogeneum est, pars autem est homogenea toti. Atque ideo non dicemus lineam minorem superficie, aut ejus partem, nec angulum contactus, partem rectilinei aut eo minorem. Si quis tamen partem latius sumat, pro omni quod quantitatem habet, et quantitatem habenti inest, poterit dicere lineam esse superficie minorem.

Things of which one is greater than, less than, or equal to the other, are homogeneous. This is obvious for equals, since they can be rendered congruent and thus also similar. The lesser is also homogeneous with the greater, since it is equal to, and thus homogeneous with, its part, and the part is moreover homogeneous with the whole. And for that reason we will not say that a curve, or its part, is less than a surface, nor that a contact angle is part of a rectilinear one, or less than it. Nonetheless, if one takes 'part' more broadly as anything that has quantity and is in [*inest*] something having quantity, then one can say that a line is less than a surface.

Pars minor est toto. Est enim aequalis parti ejus, nempe sibi ipsi.

A part is less than the whole. For it is equal to a part of it, namely to itself.

Totum est aequale omnibus partibus cointegrantibus. Coincidunt enim; vel certe si conjungantur quia totum componunt coincidentia reddentur. Adeoque et congruent.

The whole is equal to all of the co-integrant parts. For they coincide; or at least if they are conjoined, since they compose the whole, they will be rendered coincident. And so also congruent.

Pars partis est pars totius. Adeoque minus minore est minus majore. Nam parti minoris aequale est; ergo et parti majoris, parti scilicet partis majoris.

A part of a part is a part of the whole. So indeed something less than the lesser is less than the greater. For it is equal to a part of the lesser, therefore also to a part of the greater, that is, to a part of a part of the greater.

[*Verworfener Abschnitt*]

[The next portion was cancelled.]

Homologorum pro uno similium assumtorum eadem inter se

The homologous [components] to one of [two] assumed similars have ratione.

the same ratio to each other.

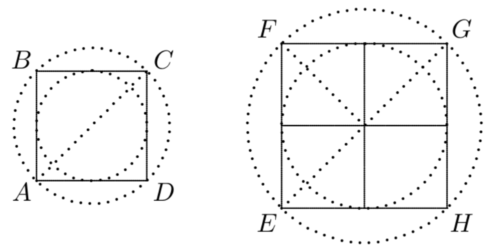

Figure 10a

Exempli gratia similia sunt quadrata *ABCD* et *EFGH*. In priore quadrato habemus latus *AB*. Ambitum *ABCDA* diagonalem *AC*. Circulum inscriptum. Circuli circumscripti circumferentiam. Aream quadrati. Aream circuli. In altero quadrato habemus respondentia: Latus *EF*. Diagonalem *EG*. Ambitum *EFGHE*. Circulum circumscriptum. Ejus aream. Quadrati ipsius aream. Possemus et utrobique Circulos inscribere, aliaque multa peragere. Hinc dico esse latus in uno ad suum diagonalem, in ea ratione in qua latus alterum etiam est ad suum diagonalem. Nam alioqui is qui in uno quadrato erit, et postea sigillatim in altero posset ea discernere, notans in uno rationem illam lateris et diagonalis esse ab ea quae est in alio diversam. Nos autem definivimus similia, quae sigillatim spectata discerni non possunt. Eandem ob causam peripheria circuli unius est ad suam diametrum, ut peripheria Circuli alterius est ad suam. Item area circuli unius est ad suae diametri quadratum; ut area circuli alterius etiam est ad suae diametri quadratum. Hinc inferre statim possemus peripherias esse inter se ut diametros; et circulos ut quadrata diametrorum. Et eodem modo Sphaeras ut diametrorum Cubos et triangulorum similium latera homologa esse proportionalia, si demonstratum jam esset rationum permutatio. Itaque ope hujus axiomatis pleraque theoremata Geometrica ab aliis

For example, the squares *ABCD* and *EFGH* are similar. In the first square, we have side *AB*, boundary *ABCDA*, diagonal *AC*. Inscribed circle. Circumference of a circumscribed circle. Area of the square. Area of the circle. In the other square we have correspondingly: Side *EF*. Diagonal *EG*. Boundary *EFGHE*. The circumscribed circle. Its area. The area of the square itself. We could also inscribe circles in either case, and carry out many other things. Hence I say that a side in the one is in the ratio to its diagonal in which also the side of the other is to its diagonal. Otherwise someone who is in the one square, and afterwards separately in the other, could distinguish them, noting that the ratio of side and diagonal in the one is different from what it is in the other. We have defined similars, though, as things which cannot be distinguished when viewed separately. For the same reason the periphery of one circle is to its diameter as the periphery of the other circle is to its. Likewise the area of one circle is to the square of its diameter as also the area of the other circle is to the square of its diameter. Hence we could immediately infer that the peripheries are to each other as the diameters, and the circles as the squares of the diameters. And in the same way spheres are as the cubes of diameters; and the homologous sides of similar triangles would be proportional if permutation of ratios were already demonstrated. And thus, with the aid of this axiom, a great many geometrical theorems, proven by others with such great effort, are demonstrated with no trouble, and we have a magno molimine comprobata, nullo negotio demonstrantur, et novum habemus principium inveniendi. Cum alias circulos esse ut quadrata diametrorum, et sphaeras ut cubos, Euclides per deductionem ad absurdum demonstrare sit coactus Archimedes autem sine demonstratione assumserit, similium figurarum centra gravitatis esse similiter posita. Quae omnia ex nostra definitione similitudinis, quae hactenus nulli quod sciam in mentem venit sponte nascuntur. Idem principium valet non in Geometria tantum, sed et in aliis omnibus, ubi quantitas et qualitas conjunguntur.

new principle of discovery. Whereas Euclid was forced to demonstrate through deduction to absurdity[8] that circles are as the squares of the diameters, and spheres as the cubes, and Archimedes, on the other hand, assumed without proof that the centers of gravity of similar figures are similarly positioned. All of these emerge spontaneously from our definition of similarity, which, as far as I know, has not occurred to anyone until now. The same principle is valid not only in geometry, but also in everything else where quantity and quality are combined.

Heterogenea autem non debent comparari inter se, neque enim ratio nisi inter homogenea est; alioqui oriretur absurdum. Exempli causa si latus *AB* esset ad sui quadrati *ABCD* aream, ut alterum latus *EF* est ad aream quadrati sui *EFGH*, tunc (per ea quae suo loco demonstrabuntur) permutando forent latera *AB* et *EF*, inter se, ut quadrata *ABCD*, et *EFGH*.

Heterogeneous things should not, however, be compared with each other, neither is there any ratio except between homogeneous things, otherwise absurdity would arise. For example, if the side *AB* were to be to the area of its square *ABCD* as another side *EF* is to the area of its square *EFGH*, then (according to what will be shown in its own place), by permutation, sides *AB* and *EF* would be to each other as the squares *ABCD* and *EFGH*.

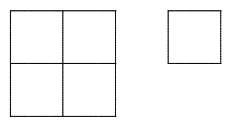

Figure 10b

Quod est absurdum, nam si exempli causa *EF* est duplum ipsius *AB* tunc quadratum ab *EF* est quadrati ab *AB* non duplum sed quadruplum. Et cubus cubi duplo longioris non duplus sed octuplus. Idem est in circulis et sphaeris, quod in quadratis et cubis.

Which is absurd; for if, for example, *EF* is the double of *AB*, then the square of *EF* is not the double but the quadruple of the square of *AB*. And the cube of something double the length is not the double but the octuple of the cube. The same for circles and spheres as for squares and cubes.

Aequimultiplorum eadem ratio est quae simplorum, patet ex his quae diximus ad definitionem proportionalium. Nam

Equimultiples have the same ratio as the simples, which is clear from what we have said in defining proportionals. For six feet are the same

[8]Latin *'deductionem ad absurdum'*, a variant of *reductio* emphasizing a movement forwards which may not simplify things.

sex pedum ad tres eadem ratio quae duorum tripodum ad unum. Quoniam idem est tripus quod tres pedes. Eorundem autem eadem ratio est, et si diversimode enuntientur, prout alia atque alia unitas seu Mensura primaria assumitur.

ratio to three which two triple-feet are to one. Since the triple-foot is the same as three feet. Moreover, their ratio is also the same when they are expressed in various ways according as one or another unit or primary measure is assumed.

Aequidivisorum eadem ratio est, quae integrorum. Nam integra sunt aequimultipla aequidivisorum.

Equidivisors have the same ratio as the whole quantities [*integrorum*]. For the wholes are equimultiples of equidivisors.

Hinc simul aequimultiplorum et aequidivisorum eadem ratio est. Sint duo A et B. erit dupli A ad duplum B eadem ratio quae A ad B. Item tertiae partis A ad tertiam partem B, eadem ratio quae A ad B, et duarum tertiarum partis A ad duas tertias ipsius B eadem ratio quae A ad B.

Hence simultaneous equimultiples and equidivisors have the same ratio. Let there be two, A and B; double A will have the same ratio to double B as A to B. Likewise the third part of A will have the same ratio to the third part of B as A to B, and two thirds part of A will have the same ratio to two thirds of B as A to B.

Ratio rationi componitur, si, antecedens ducatur in antecedentem, consequens in consequentem, ratio factorum dicitur composita simplicium, ita ratio areae rectangulae unius ad aliam est in ratione composita longitudinum et latitudinum. Hinc rationum compositio est rationis per rationem multiplicatio.

A ratio is composed with a ratio when the antecedent is multiplied by the antecedent and the consequent by the consequent; the ratio of products is called the composite of the simple ones; thus the ratio of one rectangular area to another is in the composite ratio of the lengths and widths. Hence composition of ratios is multiplication of ratio by ratio.

Ratio composita dicitur A ad C ex ratione A ad B et B ad C. Hinc ratio facti ex ductu A in B, ad factum ex ductu B in C, composita est ex rationibus A ad B et B ad C. Nam ratio facti ex A in B est ad factum ex B in C, ut A ad C (quia aequimultipla sunt, A in B et B in C ergo eandem habent rationem quam simpla A et C. per praecedentem) et ratio composita ex A ad B et B ad C etiam est A ad C.

The *ratio* of A to C is said to be *composed* of the ratios of A to B and B to C. Hence the ratio of the product of compounding A by B to the product of compounding B by C is composed of the ratios of A to B and B to C. For the ratio of the product of A by B to the product of B by C is as A to C (since they are equimultiples, A by B and B by C, therefore they have the same ratio as the simples A and C by the preceding) and the ratio composed of A to B and B to C also is A to C.

Hinc rationes ex iisdem compositae eaedem sunt. Sint rationes A ad B et B ad C. et ratio L ad M et M ad N. Sitque ratio L ad M eadem rationi A ad B ratio vero M ad N eadem rationi B ad C. Ergo erit ratio A ad C eadem rationi L ad N. Quia quae iisdem vel coincidentibus iisdem eodem modo

Hence ratios composed of the same things are the same. Let ratios[9] A to B and B to C be given, and the ratios L to M and M to N. And let the ratio of L to M be the same as the ratio of A to B, and the ratio of M to N the same as the ratio of B to C. Therefore the ratio of A to C will be the same as the ratio of L to N, since things coincide which are

[9]This and another *ratio* in this paragraph were not plural as they should be, but in a third case Leibniz clearly pluralized to *rationes* later, while editing the sentence.

determinantur coincidunt. Hinc sequitur etiamsi alius in una compositione esset rationum coincidentium ordo quam in alia tamen compositos coincidere.

the same or are determined in the same way by the same coincidents. Hence it follows that even if the order of the ratios of the coincidents in one composition were otherwise than in another, the composites would still coincide.

[*Fortsetzung des gültigen Textes*]

[Cancelled portion ends here.]

Duo homogenea habent communem mensuram quantumvis exacte propinquam. Ostendimus supra cum scalam explicaremus. Si duorum homogeneorum unum altero neque majus neque minus est, erit aequale. In scala supra posita, comparentur AG et AE. appliceturque AG ipsi scalae et puncto A manente incidet punctum G inter B et A posito AG esse minus quam AB. Ponamus jam demonstrari posse quod recta AG translata in AB, manente puncto A punctum G neque incidat intra E et B, neque inter A et E id est quod AG nec sit major nec minor quam AE. Utique punctum G incidet in ipsum punctum E. adeoque AG erit ipsi AE aequalis. Quod de duabus rectis, idem demonstrari potest de omnibus homogeneis nam omnia possunt reddi similia, et ubi similia reddita sunt, si nec magnitudine differunt nullo modo per se discerni poterunt, sed congrua erunt, adeoque cum congrua reddi possint, aequalia sunt.

Two homogeneous things have an approximate common measure of any desired precision. We showed this above while explaining the scale. If one of two homogeneous things is neither greater nor less than the other, it is equal. In the scale supposed above [Figure 8], let AG and AE be compared, and AG applied to the scale with point A remaining fixed; point G will fall between B and A, supposing that AG is less than AB. We now suppose it demonstrable that with the line AG translated to AB, and point A remaining fixed, point G falls neither between E and B, nor between A and E, that is, that AG is neither greater nor less than AE. Then certainly point G will fall on point E itself, and thus AG will be equal to AE. What can be demonstrated of two lines, the same can be done of any homogeneous things, for they can all be rendered similar, and when they are rendered similar, if they don't differ in magnitude, there is no way in which they could be distinguished, but rather they will be congruent, and so since they can be rendered congruent, they are equal.