G. W. Leibniz

Circa Geometrica Generalia

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Circa geometrica generalia et calculum situs seu picturam characteristicam

Observationes miscellae constituendae analysi geometricae plane novae praeludentes

(1) Punctum eorum quae in extenso sunt simplicissimum est. Hinc:

((1)) Punctum puncto simile est, $a \sim b$.

(2) Punctum puncto aequale est $a = b$.

(3) Punctum puncto congruit $a \simeq b$. Haec usum habebunt ad aliorum quae per certa puncta determinantur similitudines, aequalitates aut congruentias demonstrandas. Adde infra § 60.

(4) Punctum puncto, in quo assumitur coincidit, seu si sit b in a erit $a \infty b$. Ad hos paragraphos 1. 2. 3. 4. adde § 60 infra.

((4)) Imo generaliter quicquid in puncto situm est cum ipso puncto coincidit. Si plura puncta aliquam communem proprietatem habeant, et ideo unum quodque ex ipsis communi

Concerning General Geometry and the Calculus of Situation, or Pictorial Characteristic

Miscellaneous Observations Preliminary to Establishing a Completely New Geometrical Analysis[1]

(1) The point is the simplest of the things that are in extension. Hence:

((1)) Point is similar to point, $a \sim b$.

(2) Point is equal to point, $a = b$.

(3) Point is congruent to point, $a \simeq b$. These items will be useful for demonstrating similarities, equalities, or congruences of other things that are determined by specified points.

(4) A point coincides with a point which it is assumed to be in, or if b is in a, then $a \infty b$. See § 60 below regarding these paragraphs, 1, 2, 3, 4.

((4)) In fact, in general whatever is situated in a point coincides with the point itself. If multiple points have some common property, and for that reason each one of them is called by the common name X, then

[1] Leibniz deleted an alternate title: *On Pictorial Characteristic, or on the Representation of Figures through Symbols: Miscellaneous Observations and General Geometry*

nomine appelletur x, tunc locum omnibus communem et solis proprium appellabimus $\bar{x}$. Sive $\bar{x}$ significabit:

(5) Omne punctum x esse in $\bar{x}$, et

(6) omne punctum in $\bar{x}$ esse x.

(7) Si omne x est y, *erit* $\bar{x}$ *in* $\bar{y}$.

(8) Si $\bar{x}$ est in $\bar{y}$ omne x erit y.

(9) Si $\bar{x}$ sit in $\bar{y}$ et $\bar{y}$ sit in $\bar{x}$, tunc $\bar{x}$ et $\bar{y}$ *coincident*.

(10) Si $\bar{x}$ et $\bar{y}$ coincidunt, $\bar{x}$ erit in $\bar{y}$ et $\bar{y}$ erit in $\bar{x}$.

(11) Si A est in $\bar{x}$ et $\bar{x}$ in $\bar{y}$ erit A in $\bar{y}$. Hoc ita demonstratur. Si A est in $\bar{x}$ utique A est x (per artic. 6.) Jam cum $\bar{x}$ sit in $\bar{y}$ ex hypothesi omne x erit y (per 8). Ergo (per Logicam communem) etiam A erit y. Ergo (per 5) A erit in $\bar{y}$. Quod erat demonstrandum. Posset ita enuntiari haec propositio, continens continentis est continens contenti.

the locus common to all and proper to them alone we will call $\overline{X}$. That is, $\overline{X}$ will signify:

(5) every point X is in $\overline{X}$, and

(6) every point in $\overline{X}$ is X.

(7) If every X is Y, then $\overline{X}$ will be in $\overline{Y}$.

(8) If $\overline{X}$ is in $\overline{Y}$, every X will be Y.

(9) If $\overline{X}$ is in $\overline{Y}$ and $\overline{Y}$ is in $\overline{X}$, then $\overline{X}$ and $\overline{Y}$ will *coincide*.

(10) If $\overline{X}$ and $\overline{Y}$ coincide, $\overline{X}$ will be in $\overline{Y}$ and $\overline{Y}$ will be in $\overline{X}$.

(11) If A is in $\overline{X}$ and $\overline{X}$ in $\overline{Y}$, A will be in $\overline{Y}$. This is demonstrated thus. If A is in $\overline{X}$, certainly A is X (by item 6). Now since $\overline{X}$ is in $\overline{Y}$ by hypothesis, every X will be Y (by 8). Therefore (by common logic) A will be Y as well. Therefore (by 5) A will be in $\overline{Y}$. *Quod erat demonstrandum.* This proposition could be stated thus: what contains the container contains the contained.

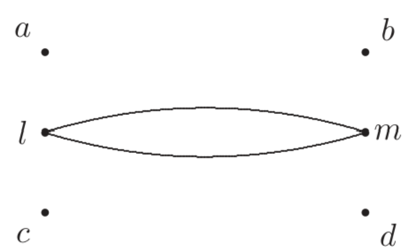

Figure 1

(12) Si idem est situs punctorum a et b inter se, qui puncto-

(12) If the situation of points a and b to each other is the same as the

rum c et d, tunc corporis rigidi puncta l et m, quae possunt applicari ipsis a et b, poterunt etiam applicari ipsis c et d.

(13) Et contra, si hoc fieri potest, idem erit situs punctorum.

(14) Idem est situs puncti a ad b, qui puncti b ad a.

(15) Si corporis rigidi puncta l et m possunt applicari punctis a et b, et quidem l ipsi a, et m ipsi b, poterunt etiam vicissim applicari l ipsi b, et m ipsi a. Nam idem est situs puncti a ad punctum b, qui puncti b ad punctum a (per 14). Ergo (per 12) fieri potest quod dictum est.

(16) Si determinata sint puncta corporis rigidi, determinati, quae data puncta simul attingere possunt, determinatus erit punctorum datorum situs inter se. De determinato adde § 25. 65.

(17) $a.b$ significat situm punctorum a et b inter se. Et $a.b.c.$ significat situm trium punctorum a et b et c inter se.

(18) Si datur $a.b.c.$ datur $a.b.$

(19) Si datur $a.b.$ et $a.c.$ et $b.c.$, datur $a.b.c.$

(20) $a.b. \simeq c.d.$ significat eundem esse situm inter puncta a et b, qui inter puncta c et d, seu rigidum aliquod intelligi posse cujus extrema sint a et b, congruum rigido cujus extrema sint c et d. Seu puncta a et b posse congruere punctis c et d, salvo situ quem a et b habent inter se.

situation of points c and d, then points l and m of a rigid body that can be attached to a and b can also be attached to c and d. See § 46.[2]

(13) And conversely, if it can be done [namely, the points can be attached, then] the situation of the points will be the same.

(14) The situation of the point a to b is the same as that of the point b to a.

(15) If points l and m of a rigid body can be attached to points a and b, and indeed l to a and m to b, then also, in reverse, l can be attached to b and m to a. For the situation of the point a to the point b is the same as that of the point b to the point a (by 14). Therefore (by 12) what was stated can be done.

(16) If, of a determined rigid body, points are determined which can touch given points simultaneously, the situation of the given points to each other will be determined. On determined things, see also § 26, 65.[3]

(17) '$a.b$' signifies the situation of points a and b between themselves. And '$a.b.c$' signifies the situation of the three points a and b and c among themselves.

(18) If $a.b.c$ is given, $a.b$ is given.

(19) If $a.b$ and $a.c$ and $b.c$ are given, $a.b.c$ is given.

(20) $a.b \simeq c.d$ signifies that the situation between points a and b is the same as that between points c and d, i.e. one can understand some rigid thing whose extremes are a and b congruent to a rigid thing whose extremes are c and d. Or, the points a and b can be made congruent [*posse congruere*] to the points c and d while preserving the

[2] Leibniz added this reference in the margin.
[3] '25, 65' in the manuscript.

situation that a and b have between themselves.

(21) Si $a.b \simeq l.m.$, et $a.c. \simeq l.n.$ et $b.c. \simeq m.n.$, erit $a.b.c. \sim l.m.n.$

(21) If $a.b \simeq l.m$, and $a.c \simeq l.n$ and $b.c \simeq m.n$, then $a.b.c \simeq l.m.n$.

(22) Si $a.b.c. \simeq l.m.n.$ erit $a.b \simeq l.m.$ et ita porro, bina binis respondentibus.

(22) If $a.b.c \simeq l.m.n$ then $a.b \simeq l.m$ and so on, pairs to corresponding pairs.

(23) Si $a.b.c. \simeq l.m.n.$ et $a.b.d. \simeq l.m.p.$ et $a.c.d. \simeq l.n.p.$ erit $a.b.c.d. \simeq l.m.n.p.$

(23) If $a.b.c \simeq l.m.n$ and $a.b.d \simeq l.m.p$ and $a.c.d \simeq l.n.p$ and $b.c.d \simeq m.n.p$,[4] then $a.b.c.d \simeq l.m.n.p$.

(24) Si duorum Extensorum communem aliquam naturam habentium puncta, quae sufficientis sint numeri pro hac natura ad certum individuum determinanda, et illa puncta eundem inter se situm habeant, in uno, quem totidem in altero; duo illa extensa inter se congrua erunt. Sit communis natura $\odot$ et ponamus determinatis quatuor punctis in $\odot$, determinatum esse individuum ipsius $\odot$, seu non nisi unicum esse $\odot$ quod eadem quatuor puncta habent, et sint duo F, et G, ex quibus tam F sit $\odot$ quam G sit $\odot$ et sint assumta quatuor puncta in F ut a, b, c, d, itemque quatuor puncta in G, ut l, m, n, p sitque $a.b.c.d \simeq l.m.n.p.$ erit $F \simeq G$. Exempli causa si sint duae circumferentiae Ellipticae et quatuor puncta in una eodem modo inter se sita sint, quo quatuor puncta in altera, tunc congruae erunt haec duae circumferentiae Ellipticae. Quia datis quatuor punctis datur ellipsis. De determinatione adde infra §. 65.

(24) If there are[5] points of two extensions having some common nature, which are of sufficient number for determining this nature down to a specific individual, and these points have the same situation among themselves in the one as the same number have in the other, then these two extensions will be congruent to each other. Let the common nature be $\odot$ and suppose that with four points determined in $\odot$, an individual of $\odot$ is determined, i.e. there is only a unique $\odot$ that has the same four points, and let there be two things F and G of which F is $\odot$ as well as G is $\odot$, and let four points be assumed in F, say a, b, c, d, and likewise four points in G, say l, m, n, p, and let $a.b.c.d \simeq l.m.n.p$; then $F \simeq G$. [[6]Or expressing it more briefly, let $\overline{a.b.c.d.F\ U}n.$ (that is, a unique F from the given $a.b.c.d$, and is of F to $a.b.c.d$) and at the same time $\overline{l.m.n.p.G\ U}n.$, and let $a.b.c.d \simeq l.m.n.p$, then $F \simeq G$.] For example, if there are two elliptical circumferences and four points on one are situated among themselves in the same way as four points on the other, then these two elliptical circumferences will be congruent. Since an ellipse is given when four points are given. On determination, see also § 65 below.

(25) Similia sunt quae separatim considerata discerni non possunt seu in quibus per se consideratis nullum notari potest

(25) Things are similar which cannot be distinguished when considered separately, or in which, considered in themselves, no differentiating

[4]Leibniz duplicated $a.c.d \simeq l.n.p$; we changed the second instance to complete the pattern.
[5]Leibniz made multiple edits and insertions in this sentence.
[6]This bracketed portion was added in the margin and later cancelled. It appears to be partially ungrammatical.

attributum discriminans, sed opus est vel ambo inter se, vel tertium aliquod utrique comparari. Ita si duae figurae sint similes nulla propositio (quae nihil forinsecus assumat) potest enuntiari de una, quae non enuntiari possit et de altera. Ut si oculus successive collocetur in duobus conclavibus ex eadem materia factis, si dissimilia sunt, notabit aliquam diversitatem, in situ atque ordine, vel etiam proportionibus partium aut linearum inter se, et angulorum cum recto comparatorum. Sed si nihil tale notari possit, tunc non habebit oculus unde alterum ab altero discernat, nisi vel ambo forinsecus simul spectet atque conferat vel aliquam mensuram (qualis mensura naturalis in homine sunt membra; imo si notabile magnitudinis discrimen sit etiam fundus oculi) secum deferat. Hinc ex. gr. duo Circuli sunt similes unumquemque enim examina separatim, duc rectas quas voles, considera angulorum rationes ad rectum et linearum rectarum rationes inter se; nihil notabis in uno quod non et in altero sis notaturus. At si duas Ellipses conferas facile notabis diversitatem. Educ enim ex centro rectam usque ad circumferentiam angulo aliquo ad axem assumto, et nota ejus rectae rationem ad axem Ellipseos, idem fac in alia Ellipsi eodem angulo, saepissime deprehendes aliam rationem; et ita facile unam ab alia discernes.

(26) Si similia sint determinantia, ipseque determinandi modus similis, etiam similia erunt determinata. De determinatione infra §. 65. 75.

(27) Hinc triangula aequiangula sunt similia, nam dato uno latere et duobus (adeoque et tribus) angulis determinatum est triangulum; si ergo anguli utrobique iidem, cum latus lateri simile sit, recta scilicet rectae, nihil apparet in determinantibus unde attributum pro uno elici possit, quod non et elici possit pro altero.

(28) Similia autem triangula habent latera proportionalia,

attribute can be noted, but one needs either to compare both with each other, or to compare some third thing with both. Thus if two figures are similar, no proposition (which assumes nothing from outside) can be stated about one that cannot also be stated about the other. So if an eye is stationed successively in two rooms made from the same material, if they are dissimilar, it will note some difference, in situation and order, or even in the proportions of parts or lines to each other, and in the angles compared with right angles. But if no such thing can be noted, then the eye will have nothing by which to distinguish the one from the other, unless it either views both simultaneously from outside and compares them, or brings with it some measure (the limbs are such a natural measure in humans; in fact even the back [*fundus*] of the eye, if there is a significant difference in magnitude). Hence, e.g., two circles are similar; for examine each separately, draw whatever lines you like, consider the ratios of angles to right angles and the ratios of straight lines to each other; you will note nothing in one that you wouldn't note in the other. But if you compare two ellipses, you will easily note a difference. For draw a line from the center out to the circumference, assuming some angle to the axis, and note the ratio of that line to the axis of the ellipse; do the same in the other ellipse at the same angle; often you will obtain a different ratio and thus easily distinguish one from the other.

(26) If the determiners are similar, and the mode of determination itself is similar, the things determined will also be similar. On determination, see also § 65, 75 below.

(27) Hence equiangular triangles are similar, for, given one side and two (and so also three) angles, the triangle is determined; if therefore the angles are the same in both, since a side is similar to a side, namely a line to a line, nothing appears in the determiners from which an attribute could be produced for one that could not also be produced for the other.

(28) Now similar triangles have proportional sides, otherwise some

alioqui notari posset aliqua laterum proportio in uno, quae non notari posset in altero. Ergo per praecedentem Triangula aequiangula habent latera proportionalia.

(29) Contra triangula quorum latera proportionalia, sunt aequiangula. Nam datis tribus lateribus triangulum est determinatum; si jam latera sint proportionalia, nullum in determinantibus, nempe lateribus, discriminans attributum reperiri potest. Ergo sunt similia; ergo utrobique eadem ratio angulorum ejusdem trianguli tum inter se, tum cum summa; summa autem angulorum utrobique eadem (facit enim duos rectos) ergo et anguli (eandem rationem utrobique habentes ad hanc summam, alioqui discrimen notari posset) utrobique iidem erunt.

(30) Quae sunt similia secundum unum determinandi modum, etiam sunt similia quoad alium determinandi modum, ita duo triangula si sint similia respectu laterum, seu habeant eandem utrobique rationem singulorum laterum ad summam laterum; erunt et similia respectu angulorum, seu habebunt eandem utrobique rationem singulorum angulorum ad summam angulorum.

(31) *Homogenea* sunt, quae vel sunt similia, vel transformatione possunt similia reddi, ut linea recta et circularis, superficies gibba et plana. Cum enim omnis linea extendi possit in rectam, omnis superficies explanari convertique in quadratum, omne solidum converti in cubum, sintque recta similis rectae, quadratum quadrato, cubus cubo, patet omnes lineas, superficies, solida inter se homogenea esse. Nam definitio Homogeneorum qua Euclides utitur huc accommodari non potest, quia ne minima quidem portio congrua potest reperiri, adeoque nec communis mensura quantumlibet exacta appropinquans. Videntur et comparari posse a causa generante, nam si duo puncta moveantur aequali celeritate,

proportion of the sides could be noted in one that could not be noted in the other. Therefore, by the preceding, equiangular triangles have proportional sides.

(29) Conversely, triangles whose sides are proportional are equiangular. For, given three sides, a triangle is determined; if the sides are already proportional, no differentiating attribute can be found in the determiners, namely the sides. Therefore they are similar; therefore there is the same ratio, in both, of angles of the same triangle to each other as well as to their sum; but the sum of the angles is the same in both (since it makes two right angles) therefore also the angles (having the same ratio to this sum in both, otherwise a difference could be noted) will be the same in both.

(30) Things that are similar according to one mode of determining are also similar with respect to another mode of determining. Thus, if two triangles are similar with respect to sides, or have the same ratio, in both, of individual sides to the sum of sides, they will also be similar with respect to angles, or will have the same ratio in both of individual angles to the sum of angles.

(31) Things are *homogeneous* which either are similar or can be rendered similar by transformation, such as a straight and a circular curve, a gibbous and a planar surface. Indeed since every curve can be extended to a straight line, every surface flattened out and converted into a square, every solid converted into a cube, and a line is similar to a line, a square to a square, a cube to a cube, it is clear that all curves, surfaces, and solids are homogeneous with each other. In fact the definition of homogeneous things that Euclid uses cannot be adapted here, since indeed one cannot find a smallest congruent portion, and so neither any common measure, however close to exact. They also seem to be comparable from the generating cause, for if two points move with equal speed and time, the curves described, though dissimilar,

et tempore, lineae descriptae licet dissimiles, tamen erunt aequales; sin eadem sit celeritas, tempus inaequale, erunt ut tempora atque ita homogenea erunt quorum ratio est. Poterit tamen Euclidaea quoque definitio huc accommodari, si curva et gibba considerentur ut polygona aut polyedra infinitangula. Adde infra §. 38.

((31)) Transformatio est mutatio quae ita fit ut simplicissima quae insunt utrobique eadem sint. Quanquam enim aliquando partes maneant, ut si quadratum mutetur in triangulum rectangulum isosceles, aliquando tamen nulla pars manet, sed puncta tantum, ut si circulus mutetur in quadratum aequale.

(32) *Aequalia* sunt, quae vel congrua sunt vel transformatione possunt congrua reddi.

(33) *Majus* est cujus pars alteri (minori) toti aequalis est.

(34) *Minus* est quod alterius (majoris) parti aequale est.

(35) Hinc demonstratur partem esse minorem toto, seu totum esse majus parte. Nam pars est aequalis parti totius (nempe sibi), ergo minor toto.

(36) Si A sit $\sqcap B$ erit $B \sqcap A$.

(37) Si quid nec majus sit nec minus, et tamen homogeneum, erit aequale. Nam cum homogeneum sit, simile reddi potest, fiat ergo simile, cumque omnia similia possunt intelligi ex se invicem fieri continuato incremento vel decremento, seu communem habere generationem, utique id quod prius generabitur crescendo (descrescendo) minus (majus) erit. Quae vero simul generabuntur erunt *aequalia,* quae propositio

will still be equal; but if the speed is the same and the time unequal, they will be [in ratio] as the times, and thus homogeneous things will be things of which there is a ratio. The Euclidean definition could also, though, be adapted here if the curved [lines] and gibbous [surfaces] were considered as polygons or polyhedra of infinitely many angles. See also § 38 below.

((31)) A transformation is a change which is made such that the simplest things that are in [*insunt*] both things are the same. For although sometimes parts remain, as when a square is changed into a right isosceles triangle, still, sometimes no part remains, but only the points, as when a circle is changed into an equal square.

(32) *Equals* are things which either are congruent or can be rendered congruent by transformation.

(33) The *greater* is that whose part is equal to another whole (the lesser).

(34) The *lesser* is that which is equal to the part of another (the greater).

(35) Hence it is demonstrated that a part is less than the whole, or the whole is greater than a part. For the part is equal to a part of the whole (namely itself), therefore is less than the whole.

(36) If A is $\sqcap B$, then $B \sqcap A$.

(37) If something is neither greater nor less, but nonetheless homogeneous, it will be equal. Indeed since it is homogeneous, it can be rendered similar, therefore let it be similar, and since all similar things can be understood to be made from each other by continuous increase or decrease, or to be generated in common, certainly that which is generated earlier in the increasing (the decreasing) will be the less (the greater). But those that are generated at the same time will

haberi poterit pro nova definitione aequalitatis. Idem tamen demonstrari poterit et ex definitione superiore, cum duo illa proposita sint similia, applicentur sibi respondentia respondentibus, tunc vel congruent et erunt aequalia, vel unum ubique excedet, alioqui non erunt similia; si enim non ubique excedet, termini eorum alicubi se se secabunt, alicubi non secabunt, quod est absurdum, nam respondentia tantum coincidere debent, sed haec, si opus est, accuratius demonstrari poterunt. Breviter quae similia sunt, non nisi magnitudine possunt discerni. Hinc concludo: si sit A non $\ulcorner B$ et A non $\urcorner B$ et A Homog. B, erit $A = B$.

(38) Si B sit in A et ambo sint homogenea nec tamen coincidant, erit A *totum, B pars*, Homogenea autem ita definienda sunt, quemadmodum supra a nobis factum est § 31, ne scilicet eorum notio totum et partem praesupponat, alioqui fit circulus.

((38)) *Partes* eiusdem totius *incommunicantes* voco, quae nullam habent partem communem. *Communicantes* quae habent.

(39) Totum et summa omnium partium incommunicantium aequantur inter se, conjungendo enim has partes inde fit totum, vel dividendo totum inde fiunt hae partes. Ergo fieri possunt coincidentia. Ergo et congrua multo magis (nam omnia coincidentia multo magis sunt congrua, sive unumquodque congruit sibi). Quae autem congrua fieri possunt, aequalia sunt.

(40) Duo coincidentia sunt congrua seu unumquodque congruit sibi. Vel in notis si $A \infty B$ erit $A \simeq B$.

(41) Quae congrua sunt, etiam aequalia sunt, si $A \simeq B$ erit

be *equals*, which proposition can be regarded as a new definition of equality. The same thing can be demonstrated also from the definition above: when two proposed things are similar, let them be applied to each other, corresponding [elements] to corresponding [elements], then either they will be congruent and equal, or one will exceed [the other] everywhere, otherwise they would not be similar; indeed, if the one does not exceed everywhere, their boundaries will cross [*secabunt*] each other somewhere and not cross somewhere, which is absurd, for only corresponding things should coincide. But these things, if needed, can be demonstrated more carefully. Briefly, things which are similar cannot be distinguished except by magnitude. Hence I conclude: if A not $\ulcorner B$ and A not $\urcorner B$ and A *homog.* B, then $A = B$.

(38) If B is in A and both are homogeneous but do not coincide, then A is the *whole* and B the *part*. But homogeneous things must be defined as we have done above in § 31, namely so that their concept does not presuppose ‘whole’ and ‘part’, otherwise it makes a circle.

((38)) I call *incommunicant* those *parts* of the same whole which have no part in common, and *communicant* those which do have [a part in common].

(39) The whole and the sum of all incommunicant parts are equal to each other. Indeed, by conjoining these parts, the whole is made [*inde fit*], or by dividing the whole, these parts are made. Therefore they can be made to coincide and therefore even more so to be congruent (for all coincidents are even more so congruent, i.e. each thing is congruent to itself); but things which can be made congruent are equal.

(40) Two coincident things are congruent, i.e. each thing is congruent to itself. Or in symbols, if $A \infty B$, then $A \simeq B$.

(41) Things which are congruent are also equal; if $A \simeq B$, then $A = B$.

$A = B$.

(42) Quae congrua sunt etiam similia sunt, si $A \simeq B$ erit $A \sim B$.

(42) Things which are congruent are also similar; if $A \simeq B$, then $A \sim B$.

(43) Quae simul similia et aequalia sunt, congrua sunt. Si $A \sim B$ et $A = B$ erit $A \simeq B$.

(43) Things which are simultaneously similar and equal are congruent. If $A \sim B$ and $A = B$, then $A \simeq B$.

((43)) *Congrua* definio quae discerni etiam collata non possunt, nisi aliis forinsecus assumtis, ut duo ova aequalia et similia non nisi situ ad externa discernentur. Hinc utique sequitur §. 41 et 43. At §. 43 ita probatur; quae aequalia sunt, congrua sunt aut talia transformatione reddi possunt §. 32. Quae vero et similia sunt transformatione opus non habent.

((43)) *Congruent* things I define as those which cannot be distinguished even when brought together [*collata*] unless something from outside is assumed, as two equal and similar eggs cannot be distinguished except by their situation to external things. From this of course § 41 and § 43 follow. But § 43 is proved thus: things which are equal are congruent or can be rendered so by transformation (§ 32), but things which are also similar have no need of transformation.

(44) Quae similia sunt, Homogenea sunt, seu si $A \sim B$ erit A Homog. B. Patet ex §. 31.

(44) Things which are similar are homogeneous, or if $A \sim B$, then A *homog*. B. This is clear from § 31.

(45) *Distantia* est minimae ab uno ad aliud lineae magnitudo, ut distantia duorum punctorum est recta; puncti a recta est perpendicularis. Eam ita exprimo: AB.

(45) *Distance* is the magnitude of the shortest curve from one thing to another, as the distance of two points is a line, and that of a point from a line is the perpendicular. I express it thus: AB.

(46) Si puncta A et B magis distant quam puncta C et D, tunc in qualibet linea ab A ad B ducta sumi potest punctum cujus idem est situs ad A (vel B), qui est situs ipsius C ad D. Quid situs, vide supra §. 12. Posset ita enuntiari, si $AB \sqcap CD$, et sit linea AXB erit aliquod punctum E, tale, ut E sit X et $AE \simeq CD$. Hoc demonstrari potest. Quia tendendo a puncto ad punctum, non potest pervenire ad majorem distantiam nisi per minorem. Nempe generaliter:

(46) If points A and B are more distant than points C and D, then on any curve drawn from A to B a point can be taken whose situation to A (or to B) is the same as the situation of C to D. What situation, see § 12 above. This could be stated thus: If $AB \sqcap CD$ and there is a curve AXB, there will be some point E such that E is X and $AE \simeq CD$. This can be demonstrated. Since, while tending from a point to a point, one cannot reach a greater distance except through a lesser. Indeed, in general:

(47) In omni continua mutatione a minori variatione pervenitur ad majorem per omnes intermedias.

(47) In every continuous change, from a lesser variation one reaches a greater through all intermediates.

(48) Omne extensum, quod partim intra partim extra aliud est, extremum ejus secat, alicubi enim incipiet in eo esse, cum paulo ante extra esset.

(49) *Secari* enim intelligitur extremum vel ambitus aliquis, ab aliquo extenso si duo puncta in extenso assumi possunt a communi concursu intervallo quantumlibet parvo distantia, quorum unum extra, alterum intra ambitum, cadit.

(50) *Tangit* quod cum ad aliquod tendat, ubi ad ipsum pervenit, iterum ab eo recedit. Itaque quod tangit aequiparari potest bis secanti, quod ubi ingressum est, rursus egreditur; et proinde secat tam in ingressu quam in egressu; momentum autem ingressus et egressus coincidere intelliguntur in contactu, et portio immersa intra ambitum censetur infinite parva.

(51) Omnis linea in se rediens, a superficie in qua ducitur partem abscindit, seu superficiem dividit in duas partes, ita ut a puncto in una parte posito, non possit duci linea ad punctum in altera positum quin lineam illam secet. Nimirum si sumatur aliqua pars extensi, et divisio sive separatio a reliquo incommunicante (seu nullam partem communem, sed tantum communem terminum habente), in ipso communi termino instituatur, necesse est separatorem ad punctum redire unde inceperat, quia punctum initiale separationis, finit cohaesionem et incipit separationem, punctum vero finale separationis finit separationem et incipit cohaesionem, seu separandum; donec scilicet initiale et finale separationis punctum coincidant.

(52) Omnis superficies integra alicujus corporis finiti, ita ipsum claudit, ut non possit a puncto extra corpus ad punctum intra corpus linea duci, quin superficiem illam secet.

(48) Every extensum which is partly inside and partly outside another crosses [*secat*] its extremity, for somewhere it begins to be inside it, while a little earlier it had been outside.

(49) For an extremity or ambit is understood to be *crossed* [*secari*] by some extensum if two points in the extensum can be taken, distant by however small an interval from the common meeting point [*communi concursu*], one of which falls outside the ambit, the other inside.

(50) A thing is *tangent* [*tangit*] which, as it tends toward something, when it reaches it, recedes from it again. And so what is tangent can be identified with something crossing twice, which, when it has entered, again exits, and hence it crosses upon entering as well as upon exiting; but the moments of entry and exit are understood to coincide in the [point of] contact, and the portion immersed inside the ambit is regarded as infinitely small.

(51) Every curve that returns to itself cuts off a part from the surface on which it is drawn, or divides the surface into two parts, so that from a point placed in one part no curve can be drawn to a point placed in the other without crossing that curve. Of course if some part of an extensum is taken, and a division or separation from an incommunicant remainder (i.e. having no common part, but only a common edge [*terminus*]) is established by the common edge itself, it is necessary that the separator return to the point from which it began, because the initial point of the separation ends the cohesion and begins the separation, whereas the final point of the separation ends the separation and begins the cohesion, or what is to be separated, namely until the initial and final point of the separation coincide.

(52) Every complete surface of a finite body encloses it such that no curve can be drawn from a point outside the body to a point inside the body without crossing that surface.

(53) Linea est via puncti, ut si punctum mobile sit X, locus ejus successivus erit linea $\overline{X}$.

(54) Superficies erit via lineae $\overline{X}$ vel $L\overline{X}M$, in priora vestigia non incidentis. Poterit designari per; $\overline{\overline{X}}$, vel per $L\overline{\overline{X}}M$.

(55) Corpus est via superficiei in priora vestigia non incidentis. Poterit designari per $\overline{\overline{\overline{X}}}$ vel per $\overline{\overline{L\overline{X}M}}$.

(56) Corpus moveri non potest, quin in priora vestigia incidat et ideo non datur alia dimensio super lineam, superficiem et corpus; scilicet in extenso, nam si praeter molem addatur potentia, ascendi potest in infinitum quod tamen nihil variat in extensione, nec novas figuras producit.

((56)) Extensum est in quo assumi possunt numero indefinita quae situm habent.

(((56))) Ea est *situs* natura, ut omnia quae habent situm ad aliqua habeant etiam situm inter se.

(57) *Punctum* est terminus lineae.

(58) *Linea* est terminus superficiei.

(59) *Superficies* est terminus corporis.

(60) *Punctum* est eorum quae in extenso sunt, seu situm habent minimum seu quod situm habet, extensionem non habet. Adde § 1. 2. 3. 4.

(61) *Spatium* est in quo per se spectato nihil aliud considerari potest quam extensio, ut locus qui manet intra vas, aqua sublata et vino substituto.

(53) A curve is the path of a point, so if a moveable point is X, its successive locus is the curve $\overline{X}$.

(54) A surface will be the path of a curve $\overline{X}$ or $L\overline{X}M$ not incident upon its previous tracks; it can be denoted by $\overline{\overline{X}}$ or by $\overline{L\overline{X}M}$.

(55) A body is the path of a surface not incident upon its previous tracks; it can be denoted by $\overline{\overline{\overline{X}}}$ or by $\overline{\overline{L\overline{X}M}}$.

(56) A body cannot move without being incident upon its previous tracks, and for that reason there is not another dimension above curve, surface, and body, namely in extension, for if besides bulk one adds power, one can ascend to infinity, which however does not vary at all in extension, nor does it produce new figures.

((56)) An *extensum* is that in which things can be assumed, indefinite in number, which have situation.

(((56))) It is the nature of *situation* that all things which have situation to something also have situation among themselves.

(57) A *point* is the extremity [*terminus*] of a curve.

(58) A *curve* is the extremity of a surface.

(59) A *surface* is the extremity of a body.

(60) A *point* is the minimum of things which are in extension, or have situation; or that which has situation but does not have extension. See § 1, 2, 3, 4.

(61) *Space* is that in which, viewed in itself, nothing else besides extension can be considered, such as the locus that remains inside a vessel when water is taken away and substituted with wine.

((61)) Spatium continuatur in infinitum neque enim ratio finium reddi potest, cum ubique uniforme sit. Spatium autem generale seu locus omnium rerum nihil aliud est quam extensum purum absolutum, seu extensum maximum, ut punctum est minimum.

((61)) Space continues to infinity, for indeed no reason can be provided for limits [*finium*] since it is everywhere uniform. But general space, or the locus of all things, is nothing other than a pure absolute extensum, or a maximal extension, as a point is a minimal one.

(62) Omnia puncta sunt in eodem spatio. Seu dari potest corpus quotcunque data puncta comprehendens.

(62) All points are in the same space. Or, a body can be given comprehending however many given points.

((62)) Puncta quaelibet situm habent inter se.

((62)) Any points whatever have a situation among themselves.

(63) A quolibet puncto ad quolibet duci potest linea.

(63) From any point to any point a curve can be drawn.

((63)) Per quolibet puncta numero finita ejusdem corporis continui duci potest linea quae ex illo corpore non egreditur.

((63)) Through however many points, finite in number, of the same continuous body, a curve can be drawn that does not leave that body.

(64) Duci potest linea transiens per puncta data quotcunque et evitans puncta data quotcunque. Quod sic demonstro. Sit corpus continens simul omnia puncta data tam attingenda quam evitanda, ex eo eximantur partes continentes puncta evitanda, tam exiguae quantum satis est ne puncta reliqua retinenda seu attingenda laedantur seu simul eximantur; cum ergo exemtis illis partibus, corpus nihilominus maneat continuum, ergo (ex §. 63) in eo per omnia puncta residua attingenda, duci potest linea, eaque in corpore manens, adeoque exemta ex corpore evitans. Quod erat faciendum.

(64) A curve can be drawn passing through however many given points and avoiding however many given points. Which I demonstrate thus. Let there be a body containing at once all the given points, those to be touched as well as those to be avoided; let parts containing the points to be avoided be removed from it, small enough that the remaining points which are to be preserved or touched are not harmed or removed at the same time; since therefore with these parts removed the body nonetheless remains continuous, therefore (from § 63) a curve can be drawn in it touching all the remaining points while remaining in that body and thus avoiding the ones removed from the body, as was to be done [*quod erat faciendum*].

(65) *Determinatum* est, quod ex quibusdam suis conditionibus positis non nisi unicum est (adde §. 16. 24. 26). Exempli causa ex datis sitibus $A.B$ (seu ipsius A ad B), $A.C, A.D, A.E$ punctum A dicetur esse determinatum, si impossibile est dari aliud punctum, quod eundem ad puncta B, C, D, E situm habeat. Hoc est si posito $A.B.C.D.E. \simeq F.B.C.D.E.$, sit $A \infty F$,

(65) Something is *determined* which is entirely unique given certain of its posited conditions (see § 16, 24, 26). For example, a point A will be said to be determined from the given situations $A.B$ (i.e. of A to B), $A.C$, $A.D$, $A.E$, if it is impossible for another point to be given that has the same situation to the points B, C, D, E. That is, if, $A.B.C.D.E \simeq F.B.C.D.E$ being supposed, $A \infty F$ holds, then A will be

erit A ex istis determinatum idque poterit exprimi: A determ. per $A.B.C.D.E.$ Ita circulus determinatus est plano et centro positione datis et radio magnitudine.

determined from them. This can be expressed: *A determ. by $A.B.C.D.E.$* Thus a circle is determined by the plane and center being given in position and the radius in magnitude.

(66) Si punctum A sit determinatum ex suo situ ad aliqua alia puncta, ut B, C, D, E tunc aliquod ex ipsis ut B similiter erit determinatum ex situ suo ad puncta A, C, D, E. Vel si aliquot puncta relationem inter se habeant talem, ut unum ex situ suo ad reliqua determinetur; etiam quodlibet aliud ex situ suo ad reliqua praeter ipsum, determinabitur.

(66) If a point A is determined by its situation to some other points, such as B, C, D, E, then any one of them, such as B, will be similarly determined by its situation to the points A, C, D, E. Or if some points have such a relation among themselves that one is determined by its situation to the remaining ones, any other will also be determined by its situation to the remaining ones besides itself.

(67) Hinc sufficit relationem determinantem punctorum ita scribere $\overline{A.B.C.D.E}$ Un. seu relationem hanc esse unicam. Quod significat unumquodque horum ex situ suo ad reliqua determinari, seu si posito $A.B.C.D.E. \simeq F.B.C.D.E.$ est $A \infty F$ etiam posito $A.B.C.D.E. \simeq A.G.C.D.E.$ erit $B \infty G$. Et ita porro de C, D, E idem locum habebit.

(67) Hence it suffices to write the determining relation of points as $\overline{A.B.C.D.E\ Un}$., i.e. that this relation is unique. This signifies that each of these is determined from its situation to the remaining ones, or if $A.B.C.D.E \simeq F.B.C.D.E$ is supposed, then $A \infty F$ holds, and also if $A.B.C.D.E \simeq A.G.C.D.E$ is supposed, then $B \infty G$ will hold, and so forth, [the point] will have the same place for C, D, E.

(68) Si determinantia sint congrua, etiam congrua erunt determinata eodem existente determinandi modo. Adde supr. artic. 24. ex. gr. duo radii circuli, ellipses generantes, duos circulos, sphaeras, sphaeroides.

(68) If the determiners are congruent, that determined will also be congruent with the mode of determining being the same. See also item 24 above; for example two radii, [two] circles, [two] ellipses, generating[7] two circles, spheres, spheroids.

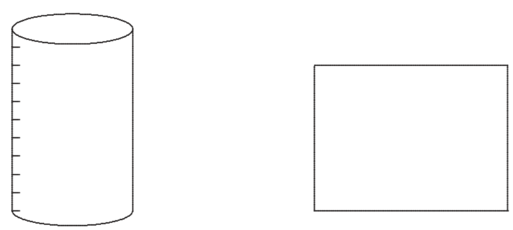

Figure 2

(69) Imo si idem sit determinandi modus, et determinantia

(69) In fact if the mode of determining is the same and the determiners

[7]The manuscript shows two abandoned attempts to indicate the generation of circles and ellipses by rotations.

sint aequalia, etiam aequalia erunt determinata; ita superficies cylindrica aequalis erit rectangulo ejusdem cum cylindro altitudinis, si basis rectanguli sit aequalis circumferentiae circuli cylindrum generantis. Nam eodem modo ex ductu rectae in altitudinem generatur rectangulum, quo ex ductu circumferentiae circuli in eandem altitudinem generatur superficies cylindrica.

are equal, the determined will also be equal; thus a cylindrical surface will be equal to a rectangle of the same altitude as the cylinder if the base of the rectangle is equal to the circumference of the circle generating the cylinder, for the rectangle is generated by compounding[8] a line by the altitude in the same way that the cylindrical surface is generated by compounding the circumference of the circle by the same altitude. [See Figure 2.]

(70) Falsum est determinata esse proportionalia determinantibus, etiamsi sit idem determinandi modus, nisi determinata sint determinantibus homogenea. Alioqui sequeretur circulos esse inter se ut radios, dato enim centro et radio determinatur circulus.

(70) It is false that determined things are proportional to the determiners, even if the mode of determination is the same, unless the determined things are homogeneous to the determiners. Otherwise, it would follow that circles are to each other as [their] radii, for, given a center and radius, a circle is determined.

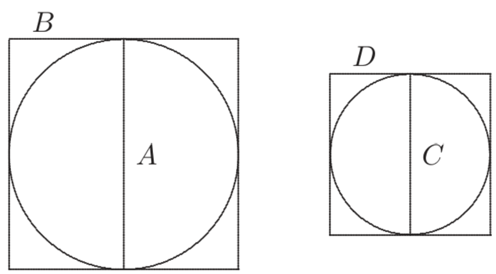

Figure 3

(71) At circulos esse ut quadrata diametrorum, quod multis ambagibus Euclides ostendit libro 12, *Elementorum*, id mihi ex definitione similitudinis primo statim obtutu patet. Nam circulus A cum quadrato circumscripto B constituit figuram similem circulo C cum quadrato circumscripto D, est enim circulus circulo similis, quadratum quadrato simile, et modus applicandi circuli ad quadratum etiam similis utrobique. *Ergo ea est ratio A ad B quae C ad D* (alioqui in $A.B$ per se spectato

(71) Rather, that circles are as the squares of [their] diameters, which Euclid showed with much circuity in Book 12[9] of the *Elements*, is immediately clear to me at the first glance from the definition of similarity. For circle A with circumscribed square B constitutes a similar figure to circle C with circumscribed square D. [See Figure 3.] For a circle is similar to a circle, a square similar to a square, and the method of adding the circle to the square is also similar in both cases. *Therefore, the ratio of A to B is the same as that of C to D* (otherwise

[8] Latin *ex ductu*.

[9] Leibniz wrote Book 10. The Academy Edition editors suggest that he intended XII.2 of Euclid's *Elements*.

observari posset aliquod discriminans a $C.D$ per se spectato; nam subtrahendo A a B, et residuum ab A quoties fieri potest, et residuum secundum a primo residuo rursus quoties fieri potest, et ita porro, discrimen in numeris substractionum possibilium observaretur operando circa figuram $A.B$ ab eo quod eveniret operando circa figuram $C.D$). Ergo invertendo *eadem quoque ratio erit* A *ad* C *quae* B *ad* D. Quod erat dem. Eadem methodo demonstratur[:]

something could be observed in $A.B$, viewed in itself, distinguishing it from $C.D$ viewed in itself; indeed subtracting A from B, and the remainder from A as many times as possible, and again the second remainder from the first remainder as many times as possible, and so on, a difference in the number of possible subtractions would be observed working with figure $A.B$ from what would result working with figure $C.D$). Therefore, by inversion, *the ratio of* A *to* C *will also be the same as that of* B *to* D. *Quod erat demonstrandum.* By the same method, one demonstrates:

(72) *Omnes superficies esse ut quadrata rectarum determinantium* et similiter[:]

(72) *all surfaces are as the squares of the determining lines*, and similarly,

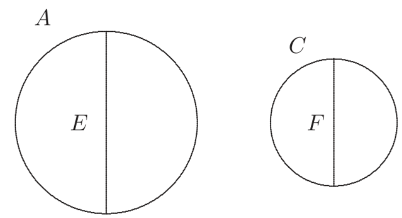

Figure 4

(73) Sphaeras vel alias *figuras solidas similes esse ut cubos rectarum determinantium.* Non autem licet dicere circulos A et C esse ut diametros E et F, licet circuli cum diametris suis etiam similes figuras utrobique constituant, cum enim nulla detur ratio circuli (superficiei) ad diametrum (lineam) quia homogenea non sunt, non potest dici esse A ad E, ut C ad F, ergo nec invertendo esse A ad C ut E ad F.

(73) spheres or other *similar solid figures are as the cubes of the determining lines.* But one may not say that circles A and C are as their diameters E and F, even if the circles with their diameters also constitute similar figures in both cases, for, since there does not exist a ratio of a circle (a surface) to a diameter (a curve) because they are not homogeneous, one cannot say A is to E as C to F, nor therefore by inversion that A is to C as E to F.

(74) Si determinantia sint similia idemque determinandi modus, etiam determinata erunt similia, adde supra §. 26. Hinc omnes circuli sunt similes inter se, item omnia quadrata: et parabola parabolae, et Ellipsis ellipsi similis est, cum latus rectum et transversum proportionalia. At linea parallela

(74) If the determiners are similar and the mode of determining is the same, the determined things will also be similar (see also § 26 above). Hence all circles are similar to each other, likewise all squares, and parabola to parabola, and an ellipse is similar to an ellipse when the latus rectum and transverse [axis] are proportional. But a curve

Ellipsi non est Ellipsis, et linea parallela parabolae non est parabola. Quaenam autem lineae parallelae sint, dicemus suo loco.

(75) Si determinantia sint coincidentia idemque determinandi modus, etiam determinata erunt coincidentia. Ita si planum et in eo centrum sint positione data, et radius magnitudine, circulus est determinatus. Si ergo in eodem plano vel in planis opinione duobus re coincidentibus duo circuli esse dicantur quorum radii aequales sint, et reperiatur eorum centra coincidere, ipsi circuli coincident.

(76) Si A sit simile, aequale, congruum, coincidens, ipsi B, et B ipsi C, erit et A ipsi C.

(77) Aequalia possunt substitui in locum aequalium salva aequalitate, seu si aequalibus addas adimasve aequalia, vel aequalia multiplices aut dividas per aequalia, prodeunt aequalia. Illud vero non sequitur, neque ex hoc nostro axiomate demonstrari potest, quaecunque in se ipsa ducta producunt aequalia, sunt inter se aequalia, nam $+3$ et -3 singula per se ipsa multiplicata, producunt 9, quae tamen aequalia non sunt, cum differentia eorum sit 6; non ergo potentiis existentibus aequalibus radices sunt aequales, etsi radicibus existentibus aequalibus potentiae sint aequales.

(78) Coincidentia possunt substitui pro his quibus coincidunt, salvis omnibus, sunt enim revera eadem, et *Eadem* definio quae sibi ubique subsituti possunt salva veritate, in propositionibus scilicet quae directae sunt nec in ipsum considerandi modum reflectuntur. Arcus circuli et curva uniformis in plano ubique sibi substitui possunt exceptis propositionibus reflexivis, qualis ista est, si quis dicat: arcus circuli concipi potest, sine ullo respectu ad planum, quanquam si quis rigorosius agere velit, defendi possit haec substitutio etiam in reflexivis.

parallel to an ellipse is not an ellipse, and a curve parallel to a parabola is not a parabola. What exactly the parallel curves are, we will say in the appropriate place.

(75) If the determiners are coincident and the mode of determining is the same, the determined things will also be coincident. Thus if a plane and a center on it are given in position, and a radius in magnitude, a circle is determined. If therefore there are said to be two circles in the same plane or in planes thought to be two but in reality coincident, whose radii are equal, and it is found that their centers coincide, then the circles themselves will coincide.

(76) If A is similar, equal, congruent, coincident, to B and B to C, then A will also be [thus] to C.

(77) Equals can be substituted in place of equals with equality preserved, i.e. if you add or subtract equals, or multiply or divide equals by equals, equals result. But it does not follow, nor can it be demonstrated from this axiom of ours, that whatever things produce equals when compounded by themselves are equal to each other, for $+3$ and -3 each produce 9 when multiplied by themselves, but they are not equal since their difference is 6; therefore it is not that when powers are equal the roots are equal, even though when roots are equal the powers are equal.

(78) Coincident things can be substituted for those to which they are coincident, preserving everything; indeed they are actually the same, and I define as the *same* those things which can be substituted everywhere preserving truth, in propositions, that is, which are direct and do not refer back to [*reflecti in*] the mode of consideration itself. An arc of a circle and a uniform curve in a plane can be substituted everywhere for each other except reflexive propositions, such as it is if someone said: ‘an arc of a circle can be conceived without any regard to a plane’, although if a more rigorous effort is requested, this

(79) Si B sit A, et C sit A, et vero B et C coincidant, seu sit $B \infty C$, dicetur esse *unum* A.

(80) Si B sit A, et C sit A, et B non sit C, nec C sit B, dicentur esse *duo* A. Sin B sit A, et C sit A et D sit A; et B non sit C neque D et C neque sit B neque D, et D non sit B neque C, dicentur esse *tria* A. Et ita porro. Et universum cum non tantum unum est A, dicuntur esse *plura.* Atque haec origo est *Numerorum;* et haec ipsa expressio in *Symbolo* Athanasii observatur, quanquam ibi usus ejus huic definitioni videatur contradicere, sed tollitur contradictio distinctione.

substitution could be defended even in reflexive ones.

(79) If B is A, and C is A, and in fact B and C coincide, or $B \infty C$, it is said to be *one* A.

(80) If B is A, and C is A, and B is not C, and C is not B, they are said to be *two* A. And if B is A, and C is A, and D is A, and B is not C and not D, and C is neither B nor D, and D is not B and not C, they are said to be *three* A. And so on. And in general when not just one is A, they are said to be *several*. And this is the origin of *numbers*; and this very expression is seen in the Athanasian Creed, although its use there seems to contradict this definition, but the contradiction is removed by a distinction.