G. W. Leibniz

Ad Calculum Situs Constituendum

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Latin text after: Vincenzo De Risi, *Geometry and Monadology* (Birkhäuser, 2007), Appendix (“Leibniz’s Writings”), Text n. 10, pp. 604–607.

Ad Calculum Situs Constituendum

Ad calculum situs constituendum utile est omnia à verbis reduci ad signa, remque eo usque produci donec habeatur Analysis, id est donec demonstrationes theorematum sine ope ingenii, certo ratiocinandi filo prodeant.

Punctum designetur per literam majusculam, velut A, vel enim si placet per literas graecas.

Locus qui plura puncta continet, vel de quo an contineat plura non determinatum est, designetur litera punctum designante, lineola superducta ut $\overline{X}$, et quodlibet punctum quod in hoc loco est dicetur X. Veluti si A sit in $\overline{X}$, aliquod X erit A.

Si A sit in $\overline{X}$ et ob id sit $A|\simeq|\overline{X}$, erit $\overline{X}$ punctum.

Si tempus T, et cuilibet T suum proprium respondeat X, erit $\overline{X}$ linea. Seu fiet $\overline{X}$ motu puncti; complura sint X seu si $X|\simeq|X$ ad T et sit $\overline{T}$ tempus erit $\overline{X}$ linea per X ad T; intelligitur cuilibet X esse proprium T, et cuilibet T proprium X. Seu X et T esse coordinata.

Hinc linea lineam secat in puncto, quia **sectio** est locus communis duorum mobilium eodem instanti, ita ut post id in-

On the Constitution of a Calculus of Situation

To establish a calculus of situation it is useful to reduce everything from words to symbols, and to continue the matter up until one has an analysis, that is, until the demonstrations of theorems come out without the aid of cleverness, by a sure thread of reasoning.

Let a point be denoted by a capital letter, such as A, or indeed by Greek letters if desired.

Let a locus which contains multiple points, or about which it is not determined whether it contains multiple points, be denoted by the letter denoting a point, with a line drawn above as $\overline{X}$, and any point which is in this locus will be called X. For instance, if A is in $\overline{X}$, some X is A.

Let A be in $\overline{X}$ and in addition let $\mathrm{A}| \simeq |\overline{X}$; then $\overline{X}$ will be a point.

If T is time and to every T there corresponds its own X, then $\overline{X}$ will be a curve. Or $\overline{X}$ will arise from the motion of a point; let there be many X or if $\mathrm{X}| \simeq |\mathrm{X}$ to T and $\overline{T}$ is time, then $\overline{X}$ will be a curve through X to T; it is understood that to each X there is its own T, and to each T its own X. Or X and T are coordinates.

Thus a curve intersects a curve in a point, because a *section* is the common locus of two mobile things at the same instant, so that after stans nihil sit ipsis mobilibus commune. Sed haec definitio non potest applicari ad sectionem in motu superficierum et corporum, quia duae lineae generantes suam quaevis superficiem, et duae superficies generantes quaevis suum corpus rari tales sunt ut ex toto vel parte congruere possint. At punctum puncto semper congruit sectionis ergo definitioni nostrae generali standum, ut sit totum commune duabus magnitudinibus partem communem non habentibus.

Si partem communem non habent duae lineae, itaque duo describentia puncta locum communem non nisi instantia habere possunt (nam ultra instantem locus est lineae pars), locus autem in instanti est punctum. Itaque non nisi punctum commune habent.

Recta commodissimè definietur linea cujus pars similis toti.

Esto recta $\overline{X}$ et sit $\overline{Y}$ talis, ut sit $\overline{Y}$ in $\overline{X}$, seu quodlibet Y sit X sitque $\overline{Y} \sim \overline{X}$ terminatam dicetur $\overline{X}$ recta. In hanc definitionem non impeditur punctum; porro hinc sequitur partes rectae similes esse inter se, cum eidem nempe toto sint similes. Sequitur et quamvis rectam produci posse. Cum enim pars producta dederit totum, consequens est etiam totum produci posse ut sit pars alterius totius.

Ex natura similitudinis consequitur rectas duas non nisi in uno puncto sibi occurrere posse. Habeant commune punctum A et inde egrediantur AX et AY et rursus concurrant in B. Moveantur puncta X et Y velocitatibus, quae sunt ut AXB ad AYB, ita concurrent in puncto B. Sit autem cujuscunque puncti motus uniformis. Cum sit $A(X) \simeq AX$ et $A(Y) \simeq AY$ et motus per $A(X)$ vel $A(Y)$ similis motui per AX, AY, atque adeò $A(X)(Y)$ et AXY determinentur similiter, erit et $A(X)(Y) \simeq AXY$. Cum ergo non coincidant X et Y, neque

that instant there is nothing common to those mobile things. But this definition cannot be applied to a section in the motion of surfaces and bodies, because any two curves generating their own surface and any two surfaces generating their own body are rarely such that they can be congruent in whole or in part. But a point is always congruent to a point, hence to stand for our general definition of section, that it is the whole common to two magnitudes not having a common part.

If two curves do not have a common part, and so the two points describing them cannot have a common locus except in an instant (for a locus beyond an instant is a part of a curve), but the locus in an instant is a point. And they do not have anything in common but a point.

A *straight line* will be most conveniently (best) defined as a curve of which a part is similar to the whole.

Let there be a line $\overline{X}$ and let $\overline{Y}$ be such that $\overline{Y}$ is in $\overline{X}$, or every Y is X, and let $\overline{Y} \sim \overline{X}$; $\overline{X}$ will be called a bounded line. Points do not enter into this definition; further from this it follows that parts of a line are similar to each other, namely since they are similar to the same whole. It also follows that every line can be extended. Indeed, since the extension of a part has given the whole, a consequence is that the whole can also be extended so that it is the part of another whole.

From the nature of similarity it follows that two lines cannot meet each other except in one point. Let them have common point A and let AX and AY go out from it and again come together at B. Let X and Y move with velocities which are as AXB to AYB, so they will meet in point B. But let the motion of each point be uniform. Since $A(X) \simeq AX$ and $A(Y) \simeq AY$ and the motion through $A(X)$ and $A(Y)$ is similar to the motion through AX, AY, and so $A(X)(Y)$ and AXY are determined similarly, we will have also $A(X)(Y) \simeq AXY$. Therefore since X and Y do not coincide, neither will (X) and (Y) coincide, and so neither etiam coincident (X) et (Y) adeoque nec poterit dari punctum B.

will it be possible for the point B to exist.

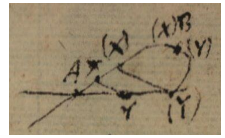

Figure 1

Hinc datis duobus punctis determinata est recta, quae puncta connectit; seu rectae extremis congruentes totae congruent.

Hence given two points, a line is determined which connects the points; or lines congruent at the extremes are congruent in whole.

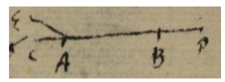

Figure 2

Et datis duobus punctis determinata est recta infinita transiens per duo puncta. Nam determinata est recta *AB*, nec produci potest utrinque nisi uno modo, ut versus *C* vel *D*, nam si ex *A* versus *C* et *E* produci posset, rectae *EAB* et *CAB* darentur, plus quam punctum commune habentes.

And given two points an infinite line passing through the two points is determined. Indeed, the line *AB* is determined, and it can only be extended in one way in either direction, as toward *C* or *D*, since if it could be extended from *A* toward *C* and *E*, lines *EAB* and *CAB* would be given which have more than a point in common.

Duae rectae *AB* et *CD* sunt similes inter se: nam *AB*, pariter ac *CD*, determinatur ex eo ut duo puncta connectuntur per rectam cujus pars sit similis toti. Itaque una determinatio alteri est similis, quare etiam similia sunt determinata, rectae *AB* et *CD*.

Two lines *AB* and *CD* are similar to each other: Indeed, *AB*, as well as *CD*, is determined from the fact that the two points are connected through the line, a part of which is similar to the whole. And so the one determination is similar to the other, for which reason the things determined, the lines *AB* and *CD*, are also similar.

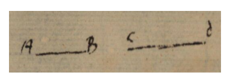

Figure 3

Hinc duae rectae aequales sunt congruae.

Hence two equal lines are congruent.

Punctum in recta est suae relationis ad duo puncta in eadem recta unicum. Esto punctum aliud D, quod eodem modo se habeat ad A et B ut C, ergo perveniri poterit ad D, ut ad C, respectu A et B, ducta recta per A et B, ad partes B perveniri poterit ad D. Contra id quod de recta est ostensum.

A point on a line is unique of its relation to two points on the line. Let there be another point D, which relates in the same way to A and B as C does, therefore it is possible to arrive at D as at C with respect to A and B, by a line through A and B, facing toward [*ad partes*] B it is possible to arrive at D. Contrary to what has been shown about the line.

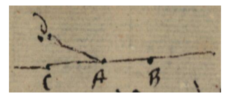

Figure 4

Puncta autem eandem relationem habentia, sunt quae eodem modo ex punctis ad quae eandem relationem habent determinari possunt. Et quae si determinari non possunt, nec possunt habere eandem relationem.

Now points having the same relation are those which can be determined in the same way from the points to which they have the same relation. And if they cannot be determined, neither can they have the same relation.

Hinc si $A.B.X$ unic. erit $\overline{X}$ recta.

Hence if $A.B.X$ is unique, then $\overline{X}$ will be a line.

Si binae rectae concurrant, nec referat quantae sint rectae, concursus **angulus** appellatur.

If two lines meet, and it does not matter how large the lines are, the meeting will be called an *angle*.

Itaque si binae rectae concurrentes congruae sint aliis binis rectis concurrentibus, modo quantum satis producantur Angulus dicitur **aequalis**.

And so if two lines that meet are congruent to another two lines that meet, provided they are extended far enough, the angle will be said to be *equal*.

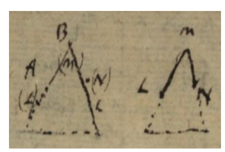

Figure 5

Itaque data sit recta AB, datoque angulo ABC, datur recta

And so if a line AB is given, with the angle ABC given, an indefinite indefinita per C, quae angulum datum facit ad AB. Ex his intelligitur si anguli eodem modo determinentur aequales fore, ut si data sit AB, et datum punctum C per quod ex B ducenda sit recta determinatur angulus.

line is given through C, which makes the given angle to AB. From this one understands that if equal angles are determined in the same way, it will happen that if AB is given and a point C is given through which a line from B is to be drawn, the angle is determined.

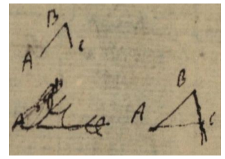

Figure 6

Hinc datis positione duobus trianguli lateribus, AB, AC, datur Angulus, ad B vel ad C.

Hence, with two sides AB, AC, of a triangle given in position, the angle to B or to C is given.

Sed ut angulus alii major minorque intelligatur, id unde sumatur Euclides non explicato. Mens tamen eius haec esse videtur, ut angulus in plano consideret et quantitatem definiat parte plani quam abscindit rectis BA, BC, infinitè productis: quod pars quam abscindit DBC intra priore cadat, Angulus est pars prioris. Sed ita opus erit explicare prius naturam plani.

But Euclid does not explain from whence he gathers that the angle of something is understood to be greater or less. However, his intention seems to be this, that an angle is considered in a plane and defines a quantity by the part of the plane which it cuts off by the lines BA and BC extended infinitely: Since the part cut off by DBC falls inside the previous one, the angle is a part of the previous one. But then it will be necessary to first explain the nature of the plane.

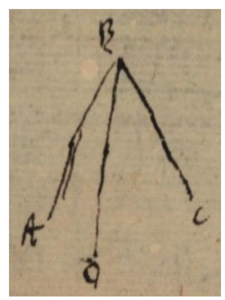

Figure 7

Si rectae binae indefinitae concurrunt, omnes rectae puncta

If two indefinite lines meet, all the lines joining any two points of the quaevis duarum rectarum conjungentes cadent in planum et planum constituent. Et hinc planum erit F locus omnium punctorum ad tria data non in eandem rectam cadentia unicorum. Sumatur utcunque in recta AC punctum D, et in recta BC punctum E, ducatur recta per DE, in qua sumatur punctum F. Dico id esse unicum ad puncta A, B, C; est enim D unicum ad A et C, et E unicum ad B et C ergo $D.E$ est unic. ad A, B, C, et quod est unicum ad $D.E$ est unicum ad A, B, C.

two lines fall in a plane and make up a plane. And hence the plane F will be the locus of all points unique with respect to three given points not falling on the same line. Let a point D be taken anywhere on line AC, and a point E on line BC, let the line through DE be drawn, and on it let a point F be taken. I claim that it is unique with respect to the points A, B, C; indeed, D is unique with respect to A and C, and E is unique with respect to B and C, therefore $D.E$ is unique with respect to A, B, C, and what is unique with respect to $D.E$ is unique with respect to A, B, C.

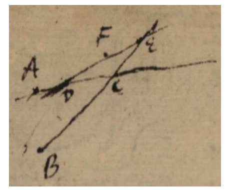

Figure 8

Sed quia videtur pleonasmus esse duas rectas quascunque puncta rectarum AC, BC, conjungentes sumimus, quia unum punctum semel occurrit, possemus sumere certa lege si placet CD, CE aequales et ita jungere quascunque DE. Optimum autem ex angulis sumere rectam. Cum recta una ad utramque alterius partem se habet eodem modo. Et ea dicetur facere priorem ad **angulos rectos**. Ita ABC et CBD sunt aequales unde et ABE et EBD sunt aequales $..ABC.. \simeq ..BDC..$ $..ABE.. \simeq ..DBC..$ Videndum an ostendi possit etiam $ABC = ABE$. Esto $ABC > ABE$ erit $ABC - ABE = DBC - DBE$, $ABC + DBE = ABE + DBE$; sed ut hinc dicamus angulus ABE esse aequalem angulo ABC opus prius est consideratione plani de qua mox. Quod si definiamus angulum rectum cum recta altera alteram secat utrinque eodem modo videndum ne sit pleonasmus, vel subreptio.

But because it seems redundant for there to be two lines, whatever lines we take joining the points of the lines AC, BC, because † *meets* one point once, we could have taken them by a certain law, CD and CE equal if desired, and thus have joined all DE. But it is best to choose a line from an angle. Since one line relates in the same way to one or the other side. And that will be said to make the previous one at right angles. Thus ABC and CBD are equal, when also ABE and EBD are equal $..ABE.. \simeq ..BDC.. \simeq ..ABE.. \simeq ..DBC$. One should see if it can be shown also that $ABC = ABE$. Let $ABC > ABE$; then $ABC - ABE = DBC - DBE$ and $ABC + DBE = ABE + DBE$; but from here in order to say that angle ABE is equal to the angle ABC there is first need for consideration of the plane, about which more soon. So if we define a right angle as when a one line intersects another in the same way on both sides, one should see if there is no redundancy or theft.

Videtur in universum dici posse rectas concurrentes, quoad concursum habere se eodem modo et congruere AEB et BEA.

It seems that lines can be said to be meeting in general, to the extent that the meeting relates in the same way and AEB and BEA are

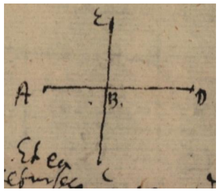

Figure 9

Videndum an ex his colligi possit congruere etiam *AEB*, et *CED*, sed res demonstrabitur mox constituta natura plani.

congruent. One should see if it can be deduced from this that *AEB* and *CED* are also congruent, but the matter will be demonstrated soon after establishing the nature of the plane.

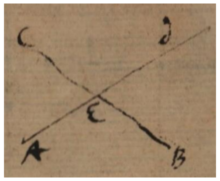

Figure 10

Ad rectam ergo *AB*, moveatur recta *CD* ita ut haec eodem suo puncto *E*, in (E) per *AB* feratur, sitque semper angulus *AEC*, angulo *CEB* aequalis, describetur planum; sed praesupponendum erit rectam *CD* inter movendum non gyrari circa *AB* velut axem.

Thus, letting line *CD* be moved toward line *AB*, in such a way that it is carried with its same point *E* to $*(E)*$ through *AB*, and letting the angle *AEC* always be equal to *CEB*, a plane will be described; but one must presuppose that, while moving, the line *CD* does not gyrate around *AB* as an axis.

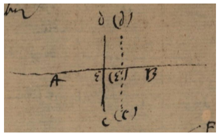

Figure 11

Hoc ergo ut ipsa constructione evitetur concipi possit rectam dari *AF* et rectam *CDE* simul super duabus rectis ferri, ita manebat in plano. Sed ubi jam volumus adhibere aliam

Therefore, for this to be avoided in the construction, one can conceive that the line *AF* is given and the line *CDE* is carried over two lines at once; thus it will remain in a plane. But when we already want to use rectam praeter AB, non est opus ipsa CE.

another line besides AB, there is no need for CE itself.

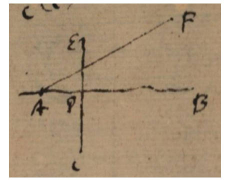

Figure 12

Itaque possumus abstinere à moto, & concipere rectam CD perpendiculanter secantem rectam AB, et sumtis punctis quotcunque in recta AB et in recta CD aequidistantibus ab E, jungere haec puncta. Rectae quaecunque tales dabunt planum quia autem CD utrinque se habet eodem plano, hoc planum à recta secatur sic ut utrinque habeat eodem modo itaque recta quaecunque planum utrinque secat eodem modo.

And so we can abstain from motion and conceive that the line CD intersects the line AB perpendicularly, and taking some number of points on the line AB and in CD equidistant from E, join these points. All such lines will give a plane, and because CD relates to the plane in the same way on both sides, this plane will be intersected by the line in such a way that it relates in the same way on both sides, and so any line intersects a plane in the same way on both sides.

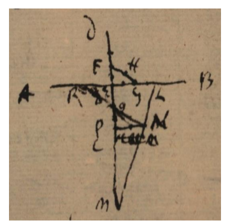

Figure 13

Hinc planum est locus punctorum, unicè se habentium ad tria puncta non in recta jacentia ACE. Nam F unice se habet ad CE, et G unice se habet ad $A.E$, ergo $F.G$ unicè se habet ad A, C, E; jam H unicè se habet ad F, G. Ergo et unicè se habet ad A, C, E. Ex hac constructione etiam planum ubique uniforme est respectu rectam AB.

Hence a plane is the locus of points relating uniquely to three points not on lying a line ACE. Indeed, F relates uniquely to CE, and G relates uniquely to CE, and G relates uniquely to $A.E$, therefore $F.G$ relates uniquely to A, C, E; now H relates uniquely to F, G. Therefore it also relates uniquely to A, C, E. [[1]From this construction the plane is also uniform everywhere with respect to the line AB.]

[1]Leibniz cancelled this sentence.

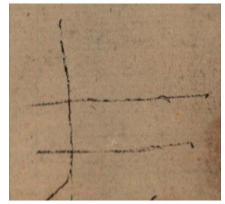

Figure 14

Sed opus esset inversione, ostendendumque planum esse locus omnium talium punctorum vel ostendendum rectam cujus duo puncta cadunt in planum etiam ipsam cadere in plano. Sumtis utcunque in rectas AB, CD punctis L, M, ostendendum est punctum rectae LM, ut N, cadere in plano. Ex puncto L agatur in CE perpendicularis NP et sumatur $PZ = PN$; juncta NZ, occurret ipsa BE in R. Et recta RZ est una ex prioribus, quae faciunt planum. Sed ostendendum erat RZ et NZ cadere in directum.

But one would need the converse, and one must show that the plane is the locus of all such points or that a line, two points of which fall in a plane, also falls in the plane itself. Taking any points L, M in the lines AB, CD, one must show that a point of line LM, say N, falls in the plane. Let NP be drawn perpendicularly from L into CE and take $PZ = PN$; NZ being joined, it will meet BE in R. And the line RZ is one of the previous ones, which make up the plane. But one needed to show that RZ and NZ fall in [the same] direction.