A category of monadic expression for Leibniz

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Abstract

I present a new formal framework for Leibniz's system of monadic expression using the language of category theory from mathematics. The basic presuppositions of this language are defended for monadic expression (namely composability and the individuation of expressings), and several of Leibniz's distinctive theses are presented in this language. These include the Identity of Indiscernibles, Universal Expression, Preestablished Harmony, as well as the thesis that every monad is characterised by its 'point of view' and that all points of view are expressions of God. A novel perspective on Leibniz's distinction between 'natural' and 'artificial' expression is given. This categorical presentation is contrasted with the currently popular 'Structural Representation' explanation of expression.

> In the same way there may be found, in one center or point, though it is perfectly simple, an infinity of angles formed by the lines which meet in it.
>
> — Leibniz

1 Introduction, purpose, plan

Leibniz's mature ontology comprised an infinity of individual substances called 'monads'. Monads are true simples with no parts.[1] Monads have qualities, and their qualities distinguish them from each other (the Principle of Identity of Indiscernibles, 'PII' below).[2] Each monad *expresses* each other monad, representing in its internal qualities the whole universe (Principle of Universal Expression, 'PUE' below).[3] This representation of multiplicity in unity he called *perception*.[4] A monad can express another monad in different ways and respects and to different degrees of distinctness; the combination of ways in which a monad

[1] "absolutely destitute of parts" L:NS, 456; cf. L:Mon, §1, 643. This isn't an exegetical work, but I'll supply some references throughout. See bibliography for citation labels.

[2] L:Mon, 643, §§8-9.

[3] L:DM, 313, §16; L:PNG, 637, §3; L:Mon, 648, §56.

[4] L, 91, note 16; L:CLA, 344; L:PNG, 636.

expresses all the others is called its *point of view*.[5] A monad is 'like a world apart', suffering no direct or causal influence from the others;[6] instead its state changes by the action of an internal principle of *appetition*.[7] Since the appetitive principle is internal, the fact that a monad expresses the universe at one time doesn't *a priori* guarantee that it will for all time. The Principle of Pre-established Harmony ('PPH') is that God arranged the original states and principles of appetition of all monads so that PUE will hold for all time,[8] and furthermore a monad's qualities are completely characterised by its perceptions, and a monad's appetition is completely characterised by its changes in perception.[9]

The concept of expression is of fundamental importance in this system. Without it there could be no difference between two monads considered separately and the same considered together; analysis of his system would end at analysis of a lone monad. And Leibniz himself told de Volder, probably hyperbolically, that his whole philosophy followed from PUE.[10]

In this essay I provide a formal framework, or logical language, for this system of mutually expressing monads. The formal framework is a *category* in the sense employed by mathematicians. (I only use 'category' with this sense below.) The centrality of expression in Leibniz's system is one motivation, but a stronger motivation for this project is simply the beauty of a parallel between Leibnizean reasoning and category-theoretic reasoning. I want to establish this parallel with some rigour.

We should require any project along these lines to satisfy a couple constraints. A logic of monadic expression must *agree with his ontology* on two basic points. It must respect the *simplicity* of monads, and it must *individuate expressions* in a way plausibly consistent with his view. Also, any such framework must reflect not only how he *explicitly* describes 'expression' (in definitions or examples), but also how he *uses* the concept, especially in his other theses. (This means it won't suffice to generalise from a few illustrative examples unless the result clarifies the meaning of, e.g., PUE.)

The main tradition in secondary literature on Leibnizean expression has come to embrace a view of expression called *structural representation* ('SR'), which says that an entity expresses another when they share structure. I hope it will become clear that this view cannot meet our constraints. Immediate (if superficial) problems are that monads aren't structured, and that expression shouldn't be individuated as a relation. (See below.) A deeper problem is that SR doesn't help us understand the use Leibniz makes of the concept in other arguments. It is too focused on 'what it is to express', in specific cases, and fails to illuminate the total system of expression considered together.

The categorical language proposed here fares better with these constraints. We find a beautiful correspondence between categorical arrows and monadic expression. This correspondence is presented in §2, and its principle interpretive commitment defended. Then in §3 the language is demonstrated by framing in its terms those theses identified above: the PII, PUE, and PPH. I also give there a novel interpretation of the distinction Leibniz makes between natural and arbitrary expression, and am able to suggest a way to understand the role of God vis-à-vis expression. In §4 the contrast with SR is given more detail.

Necessary categorical maths will be explained in the text, but I cannot expect that readers unfamiliar with categorical reasoning will readily appreciate everything.

[5] "these expressions vary in perfection as ... perspectives of the same city seen from different points" L, 269. Cf. AG, 71, 76, 143, 207, 211.

[6] L:DM, 312; L:NS, 457.

[7] L:PNG, 636.

[8] L:Mon, 648, §59; cf. also AG, 148, 195.

[9] L:PNG, 636; L:CLDV, 537.

[10] L:CLDV, 531.

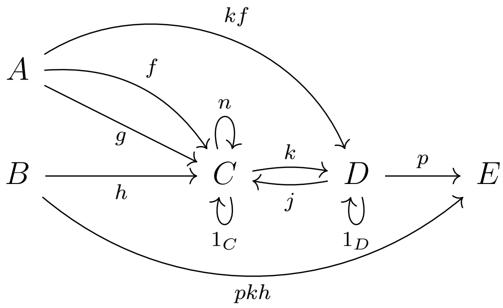


Figure 1: Illustration of a category.

2 Monadic expression as a category

A category $\mathcal{C}$ is a class of objects and a class of arrows between the objects, such that the arrows are *composable* and every object has an *identity* arrow.[11] That arrows are composable means that when an arrow ends where another begins, they can be joined into a (unique) third arrow. This head-to-tail composition must be well-defined for three or more arrows in sequence, so we require associativity: $(pk)h = p(kh) = pkh$. (Note: composition on arrow *labels* is right-left.) An identity arrow begins and ends at the same object, and does not alter arrows with which it is (pre-)composed. (This rule determines a unique identity.)

The central concepts are illustrated in Figure 1. (Some arrows are omitted for clarity.) Here, given f and k, composability implies a unique kf between A and D. The identity 1_C is such that $1_c h = h$ and $k1_c = k$. An arrow such as k has an *inverse* j if $jk = 1_C$ and $kj = 1_D$. A pair of objects sharing invertible arrows are called *isomorphic*.

Focusing on the left three objects, observe some important facts about the individuation of arrows. It is possible to have distinct arrows between the same objects, as f and g between A and C could be different. It isn't possible to have the same arrow between distinct pairs of objects; so g is certainly not identical with h. It is possible to have a variety of non-identity self-arrows, so perhaps $n \neq 1_C$.

This is the formal framework. A Leibnizean interpretation is simple to propose. 'Objects' are monads, and 'arrows' are expressions of one monad in another. In the diagram above, we'll say any of: 'h is an expressing of B in C', 'B is expressed in C by h', or $B \xrightarrow{h} C$. Let us switch now to the terms 'monad' and 'expressing' (for particulars) and 'expression' (general term). We write 'CME' for the *Category of Monadic Expression*.

In one striking passage Leibniz wrote:

> For the simplicity of a substance does not prevent the plurality of modifications which must necessarily be found together in the same simple substance; and these modifications must consist of the variety of relations of correspondence which the substance has with things outside. In the same way there may be found, in one *center* or point, though it is perfectly simple, an infinity of angles formed by the lines which meet in it.[12]

We can view this passage as foreknowledge or foreordination (as you like) of our project: we model monadic expression quite literally as (directed) lines meeting in points. (And though we don't attribute special significance to their *angles*, the arrows' mutual relationships are

[11] Mac Lane [Mac98] and Awodey [Awo10] are general references, the latter quite readable.
[12] L:PNG, 636.

essential to the categorical structure.) But to justify CME satisfactorily, each significant piece of the framework needs to be checked against Leibniz's system. Monads are certainly represented in CME as simple objects. The other significant pieces rely on an interpretive assumption. Let us consider them separately, with brief (no more than suggestive) arguments for each.

Individuation. Expressings in CME are individuated *at least* by expressing and expressed monad pairs. (So expression isn't a relation.[13]) This can be interpreted. Each expressing is always *of* a monad, *qua* unified individual substance. When $B \xrightarrow{h} C$, it is B *as a unique individual* which is (re-)presented in C's qualities (by h).

Contrast this with a model whereby C expresses B iff a part of C's set of qualities *bears some relation* to a parallel set associated with B. (As the SR view of expression would have it.) Then expression would be a relation. A relation holds between various pairs of relata. On this view there are many fewer expressings.

Let us register the question as an interpretive issue, but since the acceptability of our categorical model hinges on this question, we suggest the following argument for the first view. Leibniz thinks that perception is the representation of multiplicity (the external world) in unity (the perceiver); the expression of many in one. This implies a composition of true unities *in the representation*.[14] And of course these unities are the other monads. So an individual expressing must be *of* a monad, and plausibly of a *particular* monad.[15] We couldn't have the same expressing of two monads.

Composition. If individuation of expressings this fine-grained is the main presupposition of a categorical framework, then composability is the main supposition. Composability means that if A is expressed in B, and B in C, then A is expressed in C through an expressing determined uniquely by the first two expressings.[16] We may interpret this supposition: a monad is expressed *as a simple whole*, so any monad which expresses something must carry that expression into any other expression of itself.

Would Leibniz agree? To my knowledge Leibniz says nothing explicit about this supposition. But consider what is implied by its denial. If $A \to B$ and $B \to C$ but not $A \to C$, it seems necessary that A is expressed in a *part* of B which is disjoint from the *part* of B expressed in C. If expression is a relation between subsets of monadic qualities, this is surely possible. But if the object of an expressing is a *particular monad*, this would violate monadic simplicity. So it seems composability hangs on the same question as does individuation:

[13] Relations are *repeatable* [Mac16].

[14] I mean to parallel his argument for monads *in the world*, as necessary from multiplicity *in the world*, but apply it here within 'perception'. See also [L:Mon, 649].

[15] Maunu in [Mau04; Mau08, 254] tries to work out roughly the *opposite* view, starting from a concern that Adam's expressing 'husbandhood to Eve' makes Adam too dependent on Eve (a particular). I think he misreads Leibniz, for in [L:CLA, 335] Leibniz is *not* talking about a *particular* Adam, but a *partial* concept of Adam. A *particular* Adam, thinks Leibniz, *would* express the particular Eve, equivalently Eve's complete description, *not an incomplete description of Eve*. I believe I follow Strawson [Str64, 128] and Rutherford [Rut95, 185] on this. Maunu's whole project with 'monadic-Ramsey-ascriptions' (conjoin all true statements involving monad A and replace other monad names with existentially quantified variables) seems viciously circular. He avoids *names* for other monads but in no way writes relations in terms of *incomplete* concepts for them, since otherwise his deduction of PUE fails.
Also, if it seems hard to believe that a monad expresses the *infinite detail* of another, as required to express its particularity, recall that the *whole universe* is supposed to be deducible from every single monad's qualities.

[16] Note that since expression isn't a relation, composability isn't transitivity.

Is a (particular, complete) *monad* expressed by another (however indistinctly), or, rather, a proper subclass of a monad's qualities?

Self-expression. Self-expression is represented in CME by arrows from an object to itself. There is an important connection between self-expression and individuation. It is possible to characterise an object's self-*symmetry* in CME as its variety of non-trivial self-expressings.[17] This does not work on the coarse individuation model of SR. On that model expression is a relation, and self-expression is mere reflexivity of the relation. There is no *variety* of self-expressings.

Now every object A in a category has a unique identity self-arrow 1_A: every monad expresses itself perfectly in a special (trivial) way. (This is certainly true.) What is the identity self-expressing? It is a self-expressing satisfying at least a certain property: composition with other expressings doesn't alter those expressings. Note that this property determines a unique self-expressing *with respect to* the system of expression, i.e., the arrows: the rule 'requires' nothing of the identity expressing for a category of just one object and arrow.

Two monads A, B are isomorphic when there is a pair of expressings i, j:

$$A \underset{i}{\overset{j}{\leftrightarrows}} B$$

such that $ji = 1_A$ and $ij = 1_B$. This condition means A and B behave identically with respect to expression since every expressing of or by A or B can be written as an expressing passing through B or A (respectively). E.g. if $A \xrightarrow{f} K$, we have $f = fji : A \to B \to A \to K$.

Degrees of distinctness. We can define perfectly distinct expression of one monad by another (with respect to the whole category) in terms of perfect self-expression: D expresses C perfectly[18] via k *iff* k 'separates' incoming expressings:

$$\forall A, f, g, \text{ s.t. } A \xrightarrow{f} C \xrightarrow{k} D \text{ and } A \xrightarrow{g} C \xrightarrow{k} D,\ f \neq g \implies kf \neq kg.$$

That means nothing is 'lost' by the expressing k (with respect to the other expressings).

We might say an expressing $C \xrightarrow{k} D$ is *strictly more distinct than* an expressing $C \xrightarrow{\ell} D$ when the classes of expressings that k 'separates' for any monad, in the above sense, strictly include the classes separated by ℓ for those monads. This defines a partial order.

In the section on composability we might have worried that since expressings come in great variety, it should be possible to find two expressings (meeting head-to-tail) such that the first expresses its monad in a radically different *way* from the second, so their composition would express very little if anything at all. But (and see PUE in §3.2 for more) we can simply insist that every monad express every other at least *trivially*, and call the composition in question trivial. That is, we require existence of an expressing of each monad by each other which is less distinct than all other expressings. We can interpret this as the expression of the *mere substantial unity*, and nothing more, of the expressed monad.[19]

[17] Think of the group of symmetries of a hexagon, say, as a D_6-shaped self-expression.

[18] Such a 'monomorphism' is the generalisation of 'injection' from set-theory.

[19] Many categories in mathematics have trivial expressings; e.g. in *Group* the trivial homomorphism taking everything to the identity, or in *Topological spaces* the continuous functions taking everything to a point.

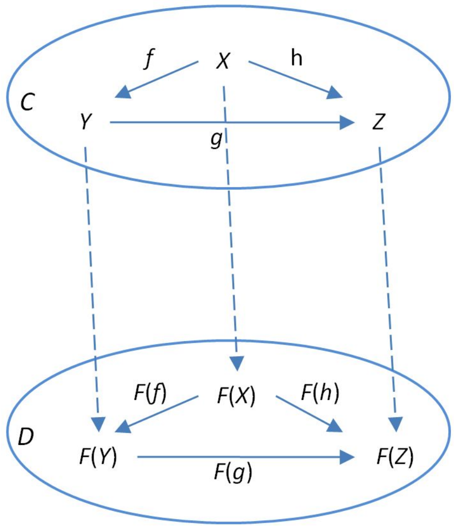


Figure 2: A functor $F : \mathcal{C} \to \mathcal{D}$. (Courtesy www.ncatlab.org.)

Functors. We will need another categorical concept to treat 'arbitrary' expression in §3.5. A category may be considered an object in the category *Cat* of all categories, and an arrow between categories in *Cat* is a *functor.* A functor $F : \mathcal{C} \to \mathcal{D}$ is a map from objects and arrows of $\mathcal{C}$ to those of $\mathcal{D}$ preserving categorical structure: (1) arrow end-points, (2) composition, i.e. $F(gf) = F(g)F(f)$ (for all f, g), and (3) identity, i.e. $F(1_A) = 1_{F(A)}$ (for all A). (See Figure 2.) This represents *expression* of one category by another.

3 Application to Leibniz's distinctive theses

If the correspondence suggested above is good, then a bare category is a *minimal* framework for monadic expression. The true test of this framework will be whether we can make sense of Leibniz's distinctive theses from within it.

3.1 Pre-established Harmony

The doctrine of Pre-established Harmony does a lot of work in the other doctrines, we shall see, so it must come first.

The doctrine has two elements: (1) the monads are in *perfect harmony*; (2) the harmony is *pre-established*, being grounded in an initial harmony together with a harmony of appetitive principles. For lack of space we cannot treat (2).

What does it mean for the monads to be in harmony? It cannot mean merely that they express each other, since that doctrine has its own title (§3.2). Rather, it means (I will suppose) that they are *completely characterised* by their expressings, taken together. Why is this harmony?

Leibniz considers in contrast to harmony the possibility that each monad, instead of relating to the others by expressing them, lives in absolute independence. It has qualities that aren't expressed in other monads. An aggregate of such monads would be *disharmonious*, we might say, in that we couldn't *make sense of them together* by understanding their mutual expression. The extent of *harmony* is the extent to which monads are characterised by their expression, and (perfect, pre-established) Harmony says that the nature of each monad is exhausted by its place in the system of expression. This is one way of interpreting Leibniz's comparison of a monad to a *mirror* that reflects the universe: a monad, like a mirror, has no 'extra' ingredients that aren't a reflection of the others.

The interpretation of Harmony in CME is that each monad is characterised by how it is expressed. More suggestively, the *situation* [situs] of a monad in the category, its *point of view* in the system, is equivalent to the monad's complete state.

This principle does not add to the categorical structure of monadic expression, but to its interpretation. It says that there is nothing in the nature of a monad that isn't 'visible to the world'; nothing that doesn't appear in the system of expression itself.

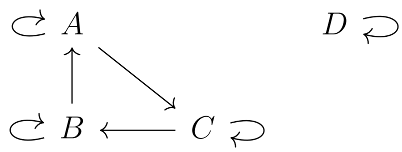
This implies a certain duality of content in CME. One can either specify the qualities of all monads and let the expressings follow from those qualities, or (assuming Harmony) one can specify all expressings, determining the monadic states by the system of expression.

Incidentally, this justifies the rules for the identity expressing and 'perfectly distinct' expression, since those were relative to (all) the other expressings.

3.2 Universal Expression

If Harmony is the doctrine that a monad is how it's expressed, Universal Expression is the dual doctrine that the universe is what each monad expresses. Harmony is expression of the one in the many; PUE is expression of the many in the one.

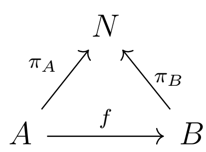
This Principle certainly doesn't hold of every category. Consider this one:



Harmony could hold (implying D is devoid of qualities other than unity). But PUE can't: the universe (specifically A, B, C) isn't expressed by D.

We could implement PUE in CME by requiring at least one arrow in each direction between every pair of objects. This could be a 'trivial' arrow, i.e. a 'least perfect' expression in the terminology of §2.

Let us instead formalise a more complex notion to capture a monad's 'point of view'. Define a *cocone*[20] to N in a category $\mathcal{C}$ as a collection of arrows, one from each object in $\mathcal{C}$, to N (the apex of this cone of arrows), such that all triangular diagrams arising this way commute. For example, if $A \xrightarrow{f} B$ is any expressing in the category, and $A \xrightarrow{\pi_A} N, B \xrightarrow{\pi_B} N$ are the 'projections' of a cocone, the triangle commutes:



This means $\pi_A = \pi_B f$, which suggests the interpretation: 'the expressing $A \xrightarrow{f} B$ is contained in the expressing π_A of A in N'. Since f was arbitrary in the category, every expressing is thus 'part of' an expressing in the top object of the cocone. Thus we can interpret a cocone as an expression of the whole category, i.e. every other monad and expressing, in the

[20] A *cone* in standard terminology has *outgoing* arrows.

one.[21] So the PUE corresponds to a requirement that cocones exist in CME. The collection of cocones to a monad may be considered its 'point of view' since it's an expression of the whole universe in the monad.

One potential objection is that Leibniz clearly thinks God should be able to recover *everything* about every other monad by looking closely enough at each, i.e. that God can see the whole universe in every monad,[22] but a generic cocone isn't enough to reconstruct the rest of the CME. (A cocone of trivial expressings is too trivial.) I cannot adequately respond but I don't feel that this objection warrants a dismissal of cocones.[23]

3.3 Identity of Indiscernibles

The famous Principle of Identity of Indiscernibles is strictly stronger than another principle which we might call Identity of Indifferents, the latter banning pairs of objects with identical qualities, and the former banning pairs merely *perceived* to have identical qualities (by any perceiver or a best perceiver).[24]

The principles come apart on a pair of monads with distinct internal qualities but which are expressed identically in all other monads. Different but indiscernible monads would be represented in CME by interpreting each object as a *collection* of monads, specifically the collection sharing the 'expressive nature' represented by the arrows to or from that object, but differing still in internal nature. The principles collapse to the same principle if we assume Harmony, since then a monad is completely characterised by the way it's expressed in others.

Assuming Harmony then, the PII is easily instituted in CME by banning isomorphic monad pairs. Then no two monads 'appear the same' with respect to the system of expression (and therefore perception).

3.4 God

Leibniz thought God is a monad in the system of monads, but with a unique role in that system.[25] I will propose here one way of construing God's unique role from within CME.

We described a monad's perspective on the universe, or point of view, as a cocone expressing all other monads (and their expressings) in one monad. But of course these cocones needn't comprise particularly *distinct* expressings; even the trivial cocone expressing nothing but unities would give a (very confused) point of view.

God's point of view should be a cocone, but it must be unique among cocones in its perfection. Every other point of view should be contained in, or subsumed by, God's.

Leibniz illustrates the difference by comparing a generic perspective to a scenograph (perspectival image of a building) and God's perspective to an ichnograph (3D 'floor plan' used by architects).[26] God's perspective is unique. From his perspective all others may be derived.

[21]This is even clearer with some technical details: a cocone is a natural transformation from the trivial diagram on $\mathcal{C}$ to the diagonal on N. A natural transformation is an 'expression' in the functor category.

[22]L:Mon, 649, §61.

[23]Maunu [Mau08, 258] also gives up this point in his attempt to explain PUE, and claims only that each monad expresses the *structure* of the universe. But I don't believe he can justify even *that* claim in his framework, since he misses the possibility of a disconnected 'D' as in the picture above.

[24]I partly follow De Risi [De 07, 384-95] in distinguishing these.

[25]That isn't entirely uncontroversial. But this isn't the place to enter the debate.

[26]R, 253; G, II.438.

The categorical concept of *colimit*[27] captures this astonishingly well. A colimit L is a 'universal cocone' in a rigorous mathematical sense. It's a cocone, with projections $\{A \xrightarrow{\pi_A} L\}_{A\in\mathcal{C}}$ from every other monad just as for another cocone, but it's unique among cocones by having the following universal property: Another cocone $\{A \xrightarrow{\gamma_A} N\}_{A\in\mathcal{C}}$ is determined by composing the cocone to L with an expression of L in N. That is, there is a unique expressing $L \xrightarrow{n} N$ such that $\forall A \in \mathcal{C}$, $\gamma_A = n\pi_A$. A natural interpretation of this is that any point of view is an expression of God's point of view,[28] and every expression of God determines a point of view.

3.5 Natural vs. arbitrary expression

Leibniz distinguished arbitrary expression from that based in nature. Illustrating the former are the characters of a written language, and the latter is a cause and its total effect.[29] We can formalise Leibniz's distinction by distinguishing two kinds of expression using categories.

(Leibniz illustrates the distinct kinds of expression with things that manifestly aren't monads, and I will do the same.)

The idea is that natural expression ('expression$_n$') is expression *simpliciter*, and is always modelled just as above in CME (perhaps without the additions of this section specific to monadic expression). Arbitrary expression ('expression$_a$') is not expression *per se* but is analogous to it: An object A expresses$_a$ another A' when A's *role* in its system of expression$_n$ parallels the *role* of A' in its own system, with the *parallel* cashed out as an expressing$_n$ of A''s system in A's system.

In categorical terms this goes as follows. Assume A is an object of $\mathcal{C}$, A' of $\mathcal{C}'$, and there is a functor between categories $F : \mathcal{C}' \to \mathcal{C}$ such that $F(A') = A$. This functor represents an expressing$_n$ of the one category in the other, according to which A' plays the role in $\mathcal{C}'$ that A plays in $\mathcal{C}$.

Notice that A and A' may be completely different kinds of thing since they are in different categories. But there is still a rule by which one can move in thought[30] from the expression to the expressed: that rule is supplied by the functor F.

One illustration of this is a novel explanation of the way an algebraic equation like

$$x^2 + y^2 = 1^2 \tag{1}$$

expresses a circle.[31] This has baffled interpreters.[32] With our categorical formalism we simply need to establish a functor between the category of algebraic equations and the category of geometric figures.[33] Such a functor would represent an expression of geometry in algebra, according to which Eqn. (1) in algebra corresponds to the circle in geometry.

[27] Again, this is dual to the *limit* for which all arrows are reversed.

[28] L, 269.

[29] L, 207-8.

[30] Leibniz mentions such a rule in [L, 207]. Brandom [Bra81, 450ff.] and Swoyer [Swo91; Swo95] emphasise the connection of expression to *reasoning* and *inference* that such a rule constitutes. CME could describe this by letting arrows represent inference possibilities.

[31] L, 207.

[32] Swoyer conjectured [Swo95, 91] that Leibniz meant the solution set of the equation (ordered pairs) expresses the circle (points in space). Puryear noticed [Pur06, 22] that the equation must also express the solution set, so Swoyer just postponed the problem. But Puryear could not explain how this happens in his SR framework since the structure of Eqn. (1) isn't shared by a circle.

[33] The modern field of algebraic geometry involves a project like this.

A more general illustration on these lines is the way a character of language expresses its referent. We'd just need a functor from a category for (phenomenal) objects in the world to a categorical model[34] of language. This would represent expression of the world in language by mapping objects to words.[35]

In both cases the symbol (word or equation) is arbitrary considered singly, but plays a definite role in a bigger structure.

4 Discussion

The Structural Representation interpretation of 'expression' has been developed by Kulstad, McRae, Swoyer, Puryear, and De Risi.[36] Kulstad's idea was that expression can be modelled as a function between sets; McRae and then Swoyer added a requirement that the function preserve some (previously specified) *structure*, making it an isomorphism. Puryear applied SR to a variety of kinds of expression. De Risi developed a more complex treatment of the 'algebra of expression' with homomorphisms instead of isomorphisms, and employed the mathematical *kernel* of Noether's homomorphism theorem to explain that part of a monad's qualities which aren't expressed distinctly.[37] All of these methods share the general strategy of explaining 'expression' as a relation holding between monads when there exists a mathematical morphism between (structured) sets of qualities associated to the monads.

There can be no doubt that Leibniz understood something close to the modern mathematical concept of structural isomorphism.[38] It would be silly to think that the SR view is *completely* misguided.

At various places we have mentioned problems for SR, but I believe it can be strengthened significantly. In the first place, if we do use structured sets of monadic *qualities*, we can overcome the problem posed to SR by monadic simplicity. (Though this involves us with a new kind of representation in need of explanation: how does a structured set represent monadic qualities?) We also discussed the problem of individuation, and I suggested that on SR expressings are individuated too coarsely. But instead of defining expression as the relation obtaining when '*there exists* a (certain) function between monadic qualities', the SR-theorist can say that an individual expressing *is* such a function. This will rectify the individuation of expressings without disturbing the rest of the SR machinery.

Unfortunately the strengthened SR isn't as strong as we should like. It emphasises the supervenience of expressings on the qualities of monads (avoiding the Scylla of the irreducible external relations Leibniz denounced), but cannot explain the way a monad's entire state is *characterised* by its expressings, in particular by its place, its 'point of view', in the system of expression (succumbing to the Charybdis of disharmony). CME does better on this front. And that's an instance of a theme: it is very hard to give an account in the SR framework of the central theses that Leibniz casts in terms of expression. We have seen how easy this is in CME. The point is that CME treats expression as a system of interest in its own right. Without such a system one cannot begin to imagine framing these other doctrines.

[34] Lam58; BOW88; Dou92; Kra03.

[35] In [L, 184] (=[G, 192]) Leibniz emphasises the 'complex mutual relation [situm complexum] or order' that the characters of language have when conjoined or inflected. Puryear [Pur06, 16-23, §2.2] takes this to be in *composite* characters, and applies the standard SR theory to explain their expression. I think Leibniz means, rather, combination in *phrase* structure because he grants 'single words' might lack such *situm*.

[36] Kul75; McR76; Kul77; Swo91; Swo95; Pur06; De 07, 321, esp. 429-36.

[37] This can be captured even more naturally in CME using a *factorisation system* [Rie08].

[38] Sti73.

This point inspires a concluding reflection. These two approaches, SR and CME, involve different *objects of explanation*. The goal of SR is to explain an instance of expression in more basic terms. It assumes a concept of *structure* (perhaps with some mathematical formalisation), and explains expression in terms of structure. Then it struggles to elucidate the system of expression as a whole. On the other hand CME leaves individual expressings unanalysed. Nothing said above helps to understand what it *means* that monad B expresses monad A by h; nothing is more basic than expressings. Perhaps this is a good thing. Leibniz occasionally gives suggestive illustrations about the meaning of expression, but he is much happier to work *with* the concept, using it to build up his framework. Perhaps we should be too, and at least for the sake of interpreting his system, develop a logic of expression that takes the concept as an irreducible element.

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