Wittgenstein's 'geometry of colour'

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Abstract

Wittgenstein's phrase 'geometry of colour' appearing in *Remarks on Colour* is explained in light of related remarks elsewhere, particularly in *Philosophical Remarks*. A traditional interpretation holds that 'geometry of colour' refers to a geometrical model of colour-space such as the colour octahedron. I argue that this interpretation is wrong, and that Wittgenstein meant by 'geometry of colour' a logico-mathematical system of colours that is analogous to our system of figures in geometry. Philosophical and textual support is given, and a formal system for colour parallel to Euclid's geometry is proposed as an illustration. The conclusion that 'geometry' meant a mathematical system rather than a colour model shows Wittgenstein in RC as stepping back in perspective from the colour octahedron, and it gives some preferential support to the 'therapeutic' rather than 'conceptual analysis' interpretation of his philosophical method.

1 Introduction

Wittgenstein asks an odd question in *Remarks on Colour:*

> "Can't we imagine certain people having a different geometry of colour than we do?" That, of course, means: Can't we imagine people having colour concepts other than ours?'[1]

Wittgenstein wrote 'geometry' only five times[2] in RC, and never explained 'geometry of colour'. I hope to explain it here. I want to develop the notion into something sharp enough to sustain its own philosophical analysis and to elicit further connections or lines of enquiry.

In earlier writings Wittgenstein discussed geometrical figures that model colour-space. These models were aimed to provide *übersichtliche Darstellungen*, or 'surveyable representations', of our colour language. In PR he spent considerable time with the colour octahedron,[3] calling it a 'rough representation of colour-space', and contrasting it with Goethe's double-cone[4] because it better reflects 'colour grammar'.[5]

[1] *Remarks on Colour* §I.66. Hereafter 'RC'; see bibliography for other citation labels.

[2] RC, §I.66, §III.35, §III.86, twice in §III.154.

[3] Put the four primary colours (RBGY) at the corners of a square. Add central points behind and before the plane of the square; label these black and white, and connect them with lines to the square's vertices. The octahedron is a *paradigm* übersichtliche Darstellung [PR, §I.1].

[4] Goethe's double-cone is the octahedron with a circle instead of the square.

[5] PR, §221ff.

There is a tradition of reading 'geometry of colour' with reference to these models so that 'geometry' signifies a spatial figure, and 'a geometry of colour' is a spatial figure representing colour grammar. The octahedron would be one example, others a rainbow, colour-wheel (Newton), or colour-sphere (Runge). One geometry of colours is compared with another, on this view, by comparing these figures.

I shall argue that this tradition is quite wrong. To show what is wrong we need to draw a sharp distinction between 'geometry of colour' as a *literal* title for a spatial model representing our colour system (which abuses somewhat the term 'geometry'), and 'geometry of colour' as a *metaphorical* reference to the colour system itself, given on account of its analogy to the system of figures in (e.g.) Euclidean geometry. One is a picture representing a system, the other *is* a system; and 'geometry' is used in very different ways. Drawing the distinction and arguing for the metaphorical interpretation are my first two tasks.

A third is to explain why this matters. Beyond the intrinsic philosophical interest of a 'geometry of colour', there is a broader interpretive significance. Understanding it correctly helps us see what Wittgenstein is doing in RC. His use of this metaphor suggests (I will argue) that he has taken *a step back* from his previous position with the octahedron (faced with a new problem he needs a new solution). Additionally, the interpretation I defend is easier to accommodate on the 'therapeutic' than the 'conceptual analysis' reading of Wittgenstein's broader methodology, though it doesn't give decisive evidence for that debate.

2 Geometry of colour

2.1

The full spectrum of colours may be smeared across a geometric figure so they blend together as their locations blend together. Smooth movements from one place to another in the figure are associated with smooth transformations between colours. Such 'blending' structure of colours is represented by any concrete model of colour-space.

Wittgenstein noticed other kinds of colour structure. The four primary colours are *different* from mixed colours: red is that colour between orange and purple which is neither yellowish nor bluish at all, but nothing analogous can be said for orange between red and yellow: there is no midpoint, no 'pure orange' that is neither reddish nor yellowish. White and black are *different:* it makes sense to blend white and black into any other colour, the others just lose 'saturation' getting lighter or darker. But (e.g.) red and green are 'opposites' and 'a reddish green blend' makes no sense.

Wittgenstein was interested in models of colour-space[6] that attempt to capture more than smooth transformations from one colour to a nearby colour, they attempt also to capture some of these other 'internal relations'[7] of colour logic. The octahedron attempts to capture the independence of white and black by adding a third dimension, and the special nature of the primaries by putting them at vertices.

In early work on RC and the 'geometry of *colour-space*',[8] Wolfgang Wenning wrote:

> What Wittgenstein sought was an ideal colour system whose relationships between colour qualities, as they are given through perception, are visualised

[6] PR, §§218-2.
[7] RC, §I.1; RFM, §104.
[8] 'Geometrie des Farbraums' [Wen82, 495]; a term Wittgenstein never used.

geometrically through distances . . . .[9]

Wenning thought that the 'Farbengeometrie' [geometry of colour] Wittgenstein had in mind was the geometry of a spatial model.

In an influential paper on RC published a decade after Wenning's, Marie McGinn also assumed that 'geometry of colour' refers to the geometrical nature of a model of colour-space. The puzzle propositions,[10] she says, 'state or describe the structural relations within the system of colour concepts that is *defined by means of the colour circle*'.[11] Another decade and Tina Wilde, in a 2003 piece exploring Wittgenstein's remark that imagining a fourth dimension for colours should be like imagining one for physical space, also assumed that these *models* are what Wittgenstein had in mind in RC: 'the octahedron or the 4th dimension — how do we connect these two colour geometries?'[12]

In a recent collection[13] devoted to RC, Richard Heinrich and Gabriele Mras concentrate expressly on colour and space, and they both take 'geometry' in the sense of spatial model. Heinrich discusses in some detail these 'geometries', i.e. these models.[14] And Mras writes:

> The problem with any claim as that a geometry of colour provides criteria for the application of colour terms is — to put it mildly — that it is not clear how such a geometry could provide criteria for the use of colour terms. How does the colour octahedron manage to "say" "that we can speak of a reddish blue but not of a reddish green etc".[15]

So a significant and continuing stream of commentators think 'geometry' refers to the geometrical figure underlying a model of colour-space.[16] Not recognising the distinction I will try to draw next, these writers didn't explicitly argue for their interpretation.

2.2

In what sense is a model of colour-space geometric? (What does *geometry* mean here?) Certainly some 'geometrical' facts about the figure in a model have counterparts in the system of colours. For instance *betweenness*: colours placed physically between two nearby points on a colour-wheel also lie between each other in a colour-sense through the possibility of 'blending' (abstractly, or with paints) and the logic of the '-ish' of colours. Or *opposites*: red lies opposite green on the colour-wheel; talk of reddish-green is like talk of a 'southwesterly northwind'.[17]

[9] Wen82, 495, my translation.

[10] E.g. 'there is transparent red but there can't be transparent white' or 'there is bluish green but there can't be reddish green'. Jonathan Westphal distilled these in early work on RC [Wes83a; Wes83b; Wes87; Wes91]; he tried to explain them by giving definitions for the internal natures of the colours following the program of Kripke and Putnam [Kri80; Put75], concluding that no 'special' colour logic is necessary. Brenner and Horner [Bre85; Hor00] defended Wittgenstein's colour logic.

[11] McG91, 444, my italics.

[12] Wil03, 285.

[13] GR14.

[14] Hei14, 42ff.

[15] Mra14, 55.

[16] See also Lugg [Lug14b, 223 note 23]. Not all take this tack. Assuming something closer to my interpretation are Brenner, Vendler, Bouveresse, Lagerspetz, and Schulte [Bre82; Bre85; Ven95; Bou04; Lag09; Sch14]. Vendler in particular, in [Ven95, §§10-11] reads 'geometry of colour' in precisely the way I do. But he doesn't recognise my distinction, the discussion is brief, and he doesn't say anything to distinguish *geometry* as an apt metaphor.

[17] RC, §I.21.

But geometry in another sense is something different, or at least much more than this; it is a logico-mathematical system: a rule-constituted game of transformations of 'ideal' shapes and figures, these having *definitions* within the game, definitions in terms of other shapes and figures. And we do not find all this in a model of colour-space. We don't find it because geometry, in this sense, is about *figures* taken together, and the logical system they comprise; there is no geometry of *a figure*. There is no geometry of a single octahedron! What's more, *one needn't be trained in geometry* to understand a concrete colour-octahedron.

Some sentences like 'this angle is wider than that angle' have a dual sense: they can express contingent, temporal propositions about concrete things ('this angle' is the angle of these spokes on this wheel[18]); or they can express atemporal, grammatical remarks, representing internal relations in the system of geometry (a 90° angle is greater than a 45° angle; couldn't be less). Similarly 'dual' propositions appear in arithmetic: 'the number of apples here is more than the number of apples there'. This can be temporal, about the apples, or atemporal, about the numbers.

Geometry is *geometry* and not arithmetic, or another system, because it is applied to figures in space. The system somehow *sublimates* from experience drawing figures with straightedge and compass: these statements go from the empirical to the grammatical without passing through a semi-empirical ('liquid') phase along the way. The system of arithmetic sublimates from counting objects, and presumably that of colour from colouring, painting, mixing, etc.

2.3

So the other side of my distinction is that we have a colour system that is analogous (at least) to the system of figures in geometry. My claim is that 'geometry of colour' is a metaphorical name for this system, not a literal term for a model.

It is immediate that the phrase *could* be used in this way: We have the same kind of dual-use sentences for colour,[19] e.g. 'this dog's colour is lighter than that one's', which may be taken to express an empirical proposition stating a fact about dogs, or an atemporal, 'grammatical' proposition about certain shades of brown in the colour system. Other sentences like 'there is no reddish green' and 'white is the lightest colour' certainly aren't empirical.[20] The systems of colour and figure have enough in common for the metaphor to go through.

It is not immediate that 'geometry of colour' *was* used this way by Wittgenstein, and less clear how far the metaphor extends if it is one. I'll try to shed more light on the latter next, and the former in §3. Regarding the latter, then, we should like to know how similar these systems are *qua* abstract calculi, but we should also like to know why geometry gives a particularly apt analogy: why should this metaphor be preferred to another?

2.4

Let us compare Euclidean geometry and a 'geometry of colour' as systems. Euclid gave five axioms:

1. There is a unique *line between* any two *points*.

[18] RFM, §I.105.

[19] RC, §I.1 and §I.32.

[20] Though Wittgenstein begins RC by noting such duality, it isn't new for him. He saw that colour relations are 'internal' already in [TLP, §4.123]. All the 'puzzle propositions' (see note 10) express atemporal norms of the system.

2. Every line segment can be *extended* to a line.

3. A *circle* may be drawn with any centre and radius.

4. All right angles are equal.

5. Parallel Postulate: If a straight line falls across two lines, the two will meet on the side with the lesser sum of interior angles, if any side's sum is lesser.

Various shapes are constructed from the primitive points, lines, and circles, and theorems proved about them. (Recall that Wittgenstein thinks such theorems are not *statements of fact* concerning abstract geometric objects, but rather further guiding norms of the system which are equivalent to their own proofs.)

Let me now propose, purely illustratively, some axioms for a colour system:

1. There are four primary colours: *red*, *yellow*, *green*, and *blue*.

2. The *lightness* of a colour is an order relation.

3. White is lighter than every colour, and black darker.

4. Any colour may be *blended* with any other to produce a third colour.

5. Blending a colour with white makes it lighter, with black, darker.

6. Reduction Postulate: Any colour may be *decomposed uniquely* into a blend of the four primaries and black and white.

7. Rules for '-ish':

    a. Arrange the primaries in a cycle RBGY. There is a *continuum* of '-ish' colours between neighbouring primaries in this cycle.
    b. Colours on each such continuum are reducible to blends of the two primaries bounding the continuum.

8. Rules for 'transparent colours':

    a. Objects appear *clearly* through transparently coloured glass.
    b. The colour of an object seen through glass appears as though blended with the colour of the glass.
    c. Objects appear darker through glass.

These axioms provide a home for the notion of 'reddish blue' and not for 'reddish green', but it can't be proved that reddish green is impossible. On the other hand, it is easy to prove that transparent white is impossible, following Wittgenstein's reasoning:[21] Any colour seen through a transparent white glass should appear as though mixed with white, and mixing with white makes every colour lighter. But everything is darker seen through any glass. Contradiction. So transparent white is impossible.

[21] In [RC, §III.191-4; see also §III.136 and 173]. Lugg explains the argument concisely in [Lug14a, 22-3]; see [Lug14b] for more complexities.

We could explore many questions here but must focus on a comparison of the systems. Both involve a small handful of primitive kinds of things, namely points, lines, and circles for the one, and primary colours, 'colourless' colours (B&W), and perhaps transparent colours for the other. Contrast the Peano axioms for arithmetic, which postulate just one primitive object '0' and construct the infinity of natural numbers by recursion. Or in axiomatic set theory (ZFC) we have just one kind of thing (set) with an infinity of token variations. So Euclidean geometry and the colour system share a certain kind of rich yet comprehensible (übersichtliche?) conceptual structure that isn't shared by other commonly employed axiom systems. This is one point of similarity.

Another point of similarity is the shared notion of 'blending' and the continuum. In Euclidean geometry a line may be drawn between any two points, and the line contains a continuum of all the points between the two. Similarly, for any two colours between a pair of primaries, there is a continuum of shades of colour between those two.

There are also differences. One such is that there is no simple correspondence between a *quantity* of colour variation between primaries and a *quantity* of space variation (length) over a line. That is to say, the *metrical* character of Euclidean geometry is not simply reflected in colour geometry. For instance it is possible in Euclidean geometry to *bisect* any line. But it is not possible, as we've remarked (p. 2), to bisect the spectrum between red and yellow to arrive at a 'pure orange'; there is no centre to that line of colours. Wittgenstein is keen on this point already in PR:

> Admittedly it's true that we can say of an orange that it's almost yellow, and so that it is 'closer to yellow than to red' and analogously for an almost red orange. But it doesn't follow from this that there must also be a midpoint between red and yellow. Here the position is just as it is with the geometry of visual space as compared with Euclidean geometry. There are here quantities of a different sort from that represented by our rational numbers.—If the expression 'lie between' on one occasion designates a mixture of two simple colours, and on another a simple component common to two mixed colours, the multiplicity of its application is different in the two cases.[22]

His first point, that lines of colour needn't have midpoints in the system of colour, is clear enough, and my point follows, that this separates colour geometry from Euclidean geometry. But what is he doing with the rational numbers? I claim this second point is to illustrate the *categorical* difference between primaries and mixed colours, as follows.

Wittgenstein distinguishes the 'geometry of visual space' from Euclidean geometry, the former being 'the syntax of the propositions about objects in visual space'.[23] A line later, the axioms of Euclidean geometry are also the 'rules of a syntax', but a different syntax. Now the system of Euclidean geometry includes all 'constructible' points on (e.g.) the real unit interval. These include the rational numbers, but also a host of irrational numbers. Rationals and irrationals are very different *kinds* of number; they have different 'multiplicity of application', as he says. But considering them as points on the interval of *visual geometry*, they appear to 'lie between' 0 and 1 in the same way; they appear to be the same kind of thing. Visual geometry makes no distinction.

The situation is similar for colours, and the similarity explains the second half of Wittgenstein's remark. The analog of 'geometry of visual space' is the syntax of colours in

[22] PR, §221.

[23] PR, §177.

colour-space. Consider the colour-wheel. (A concrete model.) There is no distinction here between the way red 'lies between' orange and purple, and the way orange 'lies between' red and yellow. But in the geometry of colour (analog of Euclidean geometry), it is clear that these colours and their 'betweennesses' are categorically different.

In sum, his remark shows a difference between the systems (midpoints in one and not the other), and brings out another analogy (categorical difference between kinds of colours and kinds of constructible points). And this further analogy also adds good evidence for my thesis: the analogy he draws emphasizes precisely the *difference* between the mathematical system of geometry (Euclid) and a more ordinary syntax of drawings in space (visual geometry). Applying the metaphor: models of colour-space represent a comparatively 'ordinary' syntax of colours, while the *real* 'geometry' of colour is a more complex, quasi-Euclidean system.

3 Wittgenstein likes a metaphor

We mentioned (note 2) all the occurrences in RC of 'geometry'. Only one is germane to our distinction. It seems to favour the metaphorical reading, though it's not about geometry 'of colour'. In §I.35 he describes Lichtenberg's 'pure white' as the construction of 'an ideal use from the actual one. The way we construct a geometry'. There is no hint of a colour-space model here, instead a similarity in *ways of construction*.

Wittgenstein compares our ability to *recognise* a primary colour to our ability to recognise a right-angle: we can 'construct' red as the colour between orange and purple but with no hint of yellow or blue, and similarly we can construct a right-angle 'by contrast with an arbitrary acute or obtuse angle'.[24] (We do not need to carry around a sample of red or of a right angle, like we would of some shade of reddish yellow or of a 31° angle.) The comparison invokes the whole system of triangles in geometry, and cannot be understood by reference to one spatial model.

In RC §I.10 he compares the request for a reddish-green to the request for a one-angled plane figure, and in §III.138 asks whether 'constructing a "transparent white body" [is] like constructing a "regular biangle"'. (Neither request makes sense in its system.) In §II.421 of RPP, written no more than a couple years before RC,[25] a comparison of 'no bluish-yellow' to 'no regular biangle' leads to: 'this could be called a proposition of colour-geometry'; and in §II.423 he calls the lack a 'geometrical gap, not a physical one'. That it's the *system* of geometry he has in mind here, and not its spatial nature or much less its application to a spatial model, is clear from the end of §II.422 where 'no regular biangle' is compared to 'no such thing as the square root of -25', and from §II.426[26] where he writes: 'We have a colour system as we have a number system.' More explicitly, in §I.624,[27] he writes: '"There's no such thing as a reddish green" is akin to the propositions that we use as axioms in mathematics.' This does not come from the arrangement of the colours at the four poles of a model, since that arrangement is arbitrary, even misleading:

> The colour-circle: the equal distances of the primary colours are arbitrary. Indeed, the transitions would perhaps make a more uniform impression on us, if,

[24] RC, §I.133.

[25] Following von Wright's appendix [PO, 486-92].

[26] Cf. also Z, §357.

[27] Z, §346.

> e.g., the pure blue were nearer to the pure green than to the pure red. It would be very remarkable if the equality of the distances lay in the nature of the things.[28]

A few remarks about 'geometry' don't involve colour but are relevant for interpreting 'geometry of colour'. He says that *arithmetic itself* is 'a more general kind of geometry' (calculations in one, constructions in the other).[29] Certainly they are compared as systems; indeed in the same sense 'chess (or any other game) is also a kind of geometry'. (A difference, we are told, is that geometry and arithmetic are 'autonomous' in their applicability, but chess isn't.) In RFM he writes on proof-theory:

> A proof of unprovability is as it were a geometrical proof; a proof concerning the geometry of proofs. Quite analogous e.g. to a proof that such-and-such a construction is impossible with ruler and compass.[30]

This is a metaphorical application of 'geometry' to something (other than colour) which isn't obviously spatial. Here there is no question of a geometric figure modelling proof-space. The point is that 'provable' is meaningful in a (background) mathematical system of the giving of certain kinds of formal proofs, and as a system it is analogous to geometry. A similar analysis applies to a remark in the Blue Book concerning the 'geometry' of interpersonal recognition,[31] and a remark in the early Cambridge lectures on 'the geometry of the process of sewing'.[32] In these contexts geometry is simply a metaphor for logic or grammar.[33]

It is relevant that Wittgenstein does not mention the colour octahedron anywhere in RC despite it being his favourite model in PR, and only mentions the colour circle (or wheel) four times in RC. He mentions geometry in *proximity* to remarks on the colour circle (RC) or octahedron (PR), but that doesn't give direct contrary evidence. With all the evidence above, we have little reason to think he meant a colour model in those places.

What is the *point* of using a metaphor? Namely geometry? Metaphors, when understood correctly, have a remarkable capacity to cut through the ambiguities of literal assertion. A metaphor makes contact with reality along a particular, unique conduit: extraneous interpretations of the literal wouldn't also 'make sense of' the metaphor. Imagine Wittgenstein had just said 'colour system', and we all agreed with him that colours have a system. But what does that *mean*? What is a 'system'? The term is so nebulous. On the other hand we all (he thought!) have a clear picture of axiomatic Euclidean geometry, and we know (even if we find it hard to *say*) what it means that it's a system. 'Geometry of colour' is far from nebulous since we have *that* system in mind. Wittgenstein wanted to invoke our knowledge of this system to make clearer his reference to 'the system' of colours.

4 Implications

I hope it is now clear that the image Wittgenstein had in mind with 'geometry of colour' was Euclidean (or similar) axiomatic geometry and not a figure like the octahedron. It is

[28]RPP, §I.623.

[29]PR, §§109-111.

[30]RFM, Appendix III §14.

[31]BLB, 61.

[32]LWL, 33-4.

[33]See also [AWL, 158]: 'The word "logical" is like the word "geometrical"', and 176: 'a proposition of geometry, i.e., of grammar'.

interesting to consider how this conclusion informs our understanding of his project in RC and his method of philosophy more broadly.

To the first question, we can say that 'geometry of colour' represents taking a *step back*, a broadening of perspective, from his previous position operating with the colour models. The octahedron 'perspicuously' (übersichtlich) presented many features of colour grammar, letting us 'see our way around' the problems of red–green immiscibility and white/black–all miscibility. It portrayed specific content about colour grammar which would be invoked by 'geometry of colour' were it used that way. We discussed critics (§2.1) who thought it invoked that content. In contrast the metaphorical 'geometry' does not carry this kind of specific content. It's a metaphor he used of different things in different contexts. It refers here to the grammar of colours, *whatever* that may be. He is looking at colours afresh, trying to find a new perspective. He is *certainly* not merely adding to the octahedron model in RC.

This conclusion is consistent with Andrew Lugg's work on the genesis of RC.[34] Lugg thinks RC was stimulated by Wittgenstein's recognition of new problems involving depth and spatiality epitomised by 'no transparent white'. Wittgenstein found a new way in which white is different, and had no 'perspicuous representation' to show him the way around depth and transparency. His old favourite, the octahedron, didn't help for the new problems, so he didn't use it; nor did he mention or refer to it.

To the second question. A central debate in Wittgenstein scholarship asks how we should reconcile apparently inconsistent themes in his work.[35] A negative theme is that philosophy, like therapy, plays the role of a doctor treating illnesses in specific people. It shows us *how to escape our philosophical traps*, and it is necessary because all of us are tempted by the bad philosophy embodied in our language. A positive theme is the power of philosophy to provide understanding by giving us *a clear overview of the structure of a conceptual system* we use. This overview lets us see our way about, not only to escape *one* trap, but to escape any number involving those concepts.

The contrast is embodied in a manner particularly relevant for this essay in two different ways of reading *übersichtliche Darstellung* (UD).[36] A reading aligning with the positive theme is that a UD is a presentation of the grammar we already know but in a form the mind can grasp as a whole, like a topographical map or model. On this view UDs are *additive* because the *understanding* of different regions of language they provide can be pasted together, like maps. And a UD is worth developing in the absence of a problem. A negative-theme reading is that a UD is *any* kind of representation that perspicuously exposes an *aspect* of a region of language, and a UD is used to dissolve a problem resulting from enslavement to *another* aspect. A UD is tailored to a specific problem, and is just a curiosity without a problem.

A negative-theme advocate can take our conclusion in stride: Wittgenstein quickly drops an old UD (octahedron) when faced with a new problem (blindness in the face of transparent white). He does not assume a particular form of the new UD he seeks, and in fact neither finds a UD nor settles on a new form. He uses axiomatic geometry as a very fine-grained *intermediate* picture,[37] providing a *system of modifications*[38] in which to go looking for a UD that resolves problems of depth and spatiality by showing a comprehensible *aspect* of these concepts.

[34] Lug14a.

[35] I draw on Kenny [Ken82] for the following characterisation.

[36] These are distinguished and the 'aspectual' negative-theme therapeutic reading promoted in Baker [Bak04]; the positive-theme 'overview of conceptual topography' reading appears in Hacker's revision of their essay on surveyability [Hac05].

[37] 'intermediate links', PI, §I.122.

[38] RPP, §I.1116; Sch14, 37-8.

This view would deny that the axiom system for colour illustrated in §2.4 is a pre-existing reality for colour concepts, and would assert rather that my presentation in §2.4 *explains the metaphor* of geometry: it shows us how to see colour *as* represented by geometry. Nothing I've written challenges these claims.

The positive-theme advocate faces two *a priori* difficulties: non-additivity and (mere) analogy. First, if UDs can be combined, why must Wittgenstein retreat from the octahedron? It's still a UD for *flat* colours. Shouldn't he have just expanded it? Second, whereas plausibly an octahedron (if that's what 'geometry' meant) represents the *actual* grammar of our language[39] and thus teaches us about our conceptual system (a positive contribution), it isn't plausible that *Euclidean* geometry (or the like) is that sort of representation; it must be mere analogy. We can add that positive-theme advocates don't have access to the concept of 'background system of modifications' to understand the role of 'geometry' in RC since they don't employ *aspects* to explain UDs (and background systems are tied to 'seeing aspects').

These difficulties are far from devastating. An interpreter committed to a positive reading might gain no advantage from this essay, but she could maintain her perspective by insisting that Wittgenstein was trying in RC to work out the actual system of colour concepts, and he would eventually have given it a UD that included the content of the old octahedron. She'd maintain that the axiom system of §2.4 is actually *true* for colour grammar, and Wittgenstein would have liked to work out its details, or else to have found a better UD. As for 'geometry of colour', she may agree that it is a metaphor as I've argued, but will not see its use as a retreat from the octahedron. The 'retreat' can be explained as an *artefact* of abandoning the traditional interpretation of 'geometry of colour' since her view will have lost the support provided by that interpretation.

In my judgement the negative 'philosophy as therapy' side accommodates the metaphorical reading more naturally, but the 'conceptual analysis' side can resist. I only hope this data triggers a Gestalt shift and the analysts recognise their need for some therapy.

[39] Eschewing Baker's arguments [Bak04] to the contrary.

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