Symmetry, Representation, and the Hole Argument

Matthew McMillan

March 2016

1

According to Einstein's General Theory of Relativity, spacetime is a smooth 4-manifold populated by several tensor fields. The metric tensor gives chronogeometric properties to spacetime and the energy-momentum tensor represents matter. Einstein's field equations link these two fields in a natural and beautiful way.

Before arriving at the generally covariant theory Einstein struggled with a 'hole argument' that seemed to imply underdetermination of these fields by his equations. The issue was revived in 1987 by Johns Earman and Norton in the context of metaphysical debate over whether spacetime is a substance. Let $\phi : M \to M$ be an automorphism of spacetime M, i.e., a diffeomorphism to itself, which is not the identity only in a small 'hole' region. All tensor fields including the metric can be pushed forward by ϕ to give new fields that agree with the old outside the hole but not everywhere inside. Thus ϕ gives rise to an isomorphism in each category of the manifold plus some of the fields, importantly (M, g, T) and $(M, \phi_* g, \phi_* T)$ are isomorphic (for metric g and energy-momentum T). Both solve thereby Einstein's equations but with different field values inside the hole, so it would seem the equations, together with superlative boundary conditions, don't determine the field values at the points of spacetime. On the other hand the two solutions *are* isometric as Lorentzian manifolds; there is no structural difference in this category.

Einstein's equations can't be faulted. Indeed in some sense this problem inevitably followed his fundamental innovation. For a pure differentiable manifold has an enormous symmetry group Diff(M), and his equations link two fields on the manifold without breaking its symmetry. Therefore a transformation of the fields which could be 'undone' by a symmetry transformation of the background smooth manifold *couldn't* change their solution status.

Philosophical discussion till recently has focused on questions of determinism and physical equivalence, identity and discernibility, and substance, essence, and accident. Recently James Weatherall approached the hole argument armed only with some doctrines about mathematical representation in physics, and Bryan Roberts has ventured a rejoinder. I would like to give what I think is a much clearer way to organize the relevant doctrines, and use it to show where I think Weatherall and Roberts go right and wrong. But in order to motivate my ideas about mathematical representation and symmetry, let me begin with a condensed tour of the more metaphysical literature, showing how the hole debate does drive the mathematical questions. I will return with applications, and if I profess one metaphysical thesis it is that we can maintain a one–one relationship between mathematical representations and physical states of affairs even in a context of symmetry.

1.1

In Earman and Norton's original paper [1] they pose the hole argument to force a dilemma between substantivalism, the view that spacetime is a substance, or at any rate a real thing, and determinism. The commitment of substantivalists to spacetime points denies them 'Leibniz Equivalence' (LE), that 'Diffeomorphic models represent the same physical situation' [1, p. 522]. Were one to *accept* LE, then 'the indeterminism discussed becomes an *underdetermination of mathematical description* with no corresponding underdetermination of the physical situation' [1, p. 524, my italics]. Their 'acid test' of substantivalism is whether the Leibnizean scenario whereby all bodies are shifted some distance gives the same or a different state of affairs.

Important to their position is that the metric is a field *on* spacetime rather than a *quality* of spacetime. (Because the metric field is allegedly dynamical and because gravitational waves allegedly carry energy.) The hole transformation moves all fields and is a kind of Leibniz shift. On these grounds it makes sense to call smooth-automorphism-generated isometries such as the hole transformation Leibniz equivalences, and a good substantivalist is thought to reject Leibniz equivalences. However there is something peculiar about 'shifting the metric field' since the metric field is the background against which motion is quantified.

1.2

Hardly a year later Tim Maudlin identified this peculiarity [2], and traced right back to Newton a view that metrical properties of spacetime points are essential. It is absurd that a *place* should be *moved*. And if 'space-time is an essentially metrical structure then the hole diffeomorphism employed by Norton and Earman, when actively interpreted, does not generate a description of a *possible* situation' [2, p. 86]. So on Maudlin's account, a spacetime point has its metrical properties in every possible world in which it appears.

We should also record his assumption that 'the diffeomorphs are separate *mathematical* objects' and 'if we fail to distinguish the representation from the objects represented we might simply accept without further examination that the diffeomorphs, which are distinct *mathematical* objects, must each correspond to a different state of affairs' [2, p. 83, his italics].

Now the trouble is that for Maudlin it's impossible for the same spacetime to have *another* metrical structure in, e.g., a non-isometric solution to Einstein's equations; unless, of course, we divide 'essence' into two kinds: one kind is reserved for possible worlds with the same metrical structure (spacetime points have metric essentialism of this kind) and another kind lives between isometry classes (they lack this kind). But how can a spacetime point tell if a local field change brings the world out of its isometry class?

Maudlin's paper opened the door to hairy questions of identity and modality. Another year and Jeremy Butterfield recognized the problem just mentioned [3, p. 20], and his response was to 'propose that we deny transworld identity to points: any point is a part of just one possible world' [3, p. 23]. This Lewisian plot requires a Lewisian solution, so Butterfield recommended using a counterpart relation for spacetime regions across worlds. Naturally he chose metric similarity, so identity is simple when regions are isometric. (And mildly less so, though trouble here isn't unique to him, when the metric also has symmetry.) Then hole transformations give 'duplicate' worlds and he is able to formulate a definition of determinism that seems to capture the 'reasons of physics' Earman demanded.

We will shortly see a striking similarity to Weatherall's proposal. Both seek to identify the globally isomorphic situations along the isomorphism. (Possible worlds for Butterfield and structures for Weatherall.) But importantly Butterfield does admit a mathematical difference, contending only for a counterpart relation that follows the metric, and Weatherall questions this mathematical difference.

If one is not prepared to accept David Lewis's picture of possibility, as I am not, then Butterfield's solution doesn't much help. But one needn't reject Lewis out of hand to acknowledge that this isn't satisfying. For the counterpart relation ends up committing one to something very *like* Leibniz Equivalence: according to the *only* inter-world relation available, hole-shifted worlds are mere duplicates, not different *because of their shift*. Furthermore his logic applies the same way to 'rigid' shifts in the symmetries of Newtonian spacetime. He can't really say that a shifted world is a *shifted* world, as others can, even if only to later explain how *shifts* in particular do nothing. Yet because points are different in each possible world, he manages to pass the 'acid test' in letter if not in spirit, calling himself substantivalist. I think the dilemma isn't resolved.

1.3

Carolyn Brighouse, Carl Hoefer, and Simon Saunders accept Leibniz Equivalence, and all three believe spacetime is real. Their challenge to Earman and Norton is with the 'acid test' itself. Earman and Norton believe: 'In sum, substantivalists, whatever their precise flavour will deny: *Leibniz equivalence* Diffeomorphic models represent the same physical situation' [1, p. 522]. On the other hand Brighouse knows that 'the hole argument is an argument against substantivalism only if the substantivalist is committed to viewing Leibniz equivalent models as representing distinct situations' [4, p. 121], yet in no uncertain terms she concludes that 'the hole argument and its ancestor the shift argument are not sufficiently strong to force a rejection of substantivalism. ... A manifold substantivalist can accept that Leibniz equivalent models represent the same situation, and thereby escape the hole argument, without any threat of being inconsistent' [4, p. 122].

Her method is one logical step from Butterfield. She thinks a substantivalist should 'individuate spacetime points or regions across possible worlds ... according to qualitative similarity' [4, p. 122]. That is, she is not afraid to identify Butterfield's duplicate counterparts. Then she fails the acid test, for the shifted (or hole-shifted) models are all equivalent. So how is she still a substantivalist? Her defense of the title is that a realist needn't be committed *a priori* to some way of identifying spacetime points across worlds, and her way is as good as any.

Brighouse's view seems to me dissatisfying in the same way as Butterfield's, and (we'll see) Weatherall's. It comes down to a fairly simple denial that the hole shifts shift anything in any sense, but without explaining how that can be. It's modal logic against metaphysics.

Hoefer and Saunders take hole transformations more seriously, and explain their metaphysical impotence along different lines. Happily the lines are quite clear. On both accounts hole transformations *do* something (or try), except that the *nature* of spacetime points is such that they are interchanged without change. Such transformations are physically meaningless.

Hoefer denies of spacetime points 'primitive identity'. A thing **A** has primitive identity just in case it *makes sense* to ask whether it's possible for **A** to exchange all its properties with another thing **B** also having primitive identity. Primitive identity implies that numerically distinct individuals could have identical properties, and is related to 'haecceitism' (that worlds can differ *de re* without differing qualitatively). Now then, says Hoefer:

> The problems about indeterminism, and essences or counterpart relations, which have dominated discussion so far all share a common, metaphysical root: the ascription of primitive identity to space-time points. I claim that this bit of metaphysics is not necessary for substantivalists, and should be eschewed. [5, p. 14]

Saunders works in a bigger and more logical framework, in which one is committed to objects one quantifies over. He shares with Leibniz the Principle of Identity of Indiscernibles (PII), but rejects Leibniz's nominalism about relations. He introduces the notion of 'weak discernibility' that lets one quantify over the *parts* of a thing having perfect symmetry. Saunders calls things indiscernible just in case they are exchangeable in every position of every n-place predication, and accepts the PII. Two things with the same properties (and not 'relatively' discernible) are 'weakly' discernible if they satisfy an irreflexive relation; e.g., being ten miles apart. So in a logical sense there are two things, weakly discernible, but switching them is meaningless. (On that he agrees with Hoefer.) Now the hole transformations are intrinsically defined symmetry transformations. Intrinsic symmetries are those 'whose physical meaning, if any, has to be expressed by [the] equations themselves' [6, p. 300].

> The consequences of the PII for such transformations are then immediate. These are intrinsically defined symmetries; they therefore leave all the real physical quantities unchanged. The world thus arrived at does not differ, in respect of any real physical property or relation, from the world with which one begins. So they are numerically the same. [6, p. 301]

I'm very sympathetic to the views of Hoefer and Saunders. I'll record that I quite like the PII, am willing to drop primitive identity and adopt weak discernibles, but am not convinced that we should reify relations (I tend closer to Leibniz than Saunders and modern logic). However my biggest concern with both projects is that the interchangeability of spacetime points (as only weakly discernible or just lacking primitive identity) seems to come somewhat apart from their role in the smooth manifold; both accounts would let one 'switch' (impotently) two *separated* points while fixing the rest of spacetime. On the view I momentarily develop the weak discernibility of points rather falls out of the *genuine diffeomorphism symmetry* of the smooth spacetime manifold. The symmetry is what counts. I'll let the entire spacetime stand as substance, but I'm not sure its 'points' are objects; if so then only in a logical sense. I think

Saunders could follow me this far, and I'd agree with him that the logical sense of object can apply here if anywhere, but I'm not sure the logical sense is the best sense.

2

Those who accept Leibniz Equivalence would identify the *a priori* distinct domain and image of hole-shift isomorphisms. To preserve spacetime substantivalism, this joins a metaphysical thesis about spacetime points to the effect that hole transformations are really meaningless.

Commentators have, as a rule, thought hole transformations are at least *mathematically* genuine; there's nothing perverse about diffeomorphisms; they really take some mathematical points to others. But this leaves a puzzle, or at least a nagging feeling something isn't right. How does the world instantiate a smooth manifold, smooth manifolds blithely admit such transformations, and yet the transformations are meaningless in the world? Shouldn't there be a fairly general thesis about mathematical representation to explain this?

Half the answer lies with the nature and number of mathematical objects, and the other half lies with the nature of symmetry.

2.1

James Weatherall is the recent exception to the rule, and in his purely mathematical solution argues 'that those alleged mathematical haecceities are spurious' [7, p. 24]. He thinks 'the mathematical argument that allegedly generates the interpretational problem is misleading' [7, p. 2]. How so? I think his stance is straightforward but let us take care to get him right.

Weatherall is strongly influenced by mathematical category theory and alleges that the hole argument rests on a category mistake. A characteristic of category theory is that objects are given, *defined* even, within types. Each type comes with a comparison (an arrow), and *only* objects of the same type can be compared directly. Contrast this with set theory, in which objects exist independently of type and one freely builds functions between types.

The category mistake is to use the *smooth manifold* comparison to present a hole transformation, and the *Lorentzian manifold* comparison to say the relevant structure is preserved. Weatherall says we can't simultaneously do both. In his words, where ψ is a hole diffeomorphism and $\tilde{\psi}$ the induced isometry:

> There is a sense in which (M, g_{ab}) and $(M, \tilde{g}_{ab})$ are the same, and there is a sense in which they are different. The sense in which

they are the same—that they are isometric, or isomorphic, or agree on all invariant structure—is wholly and only captured by $\tilde{\psi}$. The (salient) sense in which they are different—that they assign different values of the metric to the "same" point—is given by an entirely different map, namely 1_M. But—and this is the central point—one cannot have it both ways.

Key is the sense of 'sense' here. A 'sense of comparison' is an arrow in a category, relative to a type; this 'sense' is made quite rigorous in that framework. Now category theory does not naturally account for identical objects living in multiple categories, so not only are *comparisons* type-relative, but *objects themselves* are also type-relative.

We'll need a related piece of Weatherall's project. It is

> the view that mathematical models of a physical theory are only defined up to isomorphism, where the standard of isomorphism is given by the mathematical theory of whatever mathematical objects the theory takes as its models. One consequence of this view is that isomorphic mathematical models in physics should be taken to have the same representational capacities. By this I mean that if a particular mathematical model may be used to represent a given physical situation, then any isomorphic model may be used to represent that situation equally well. [7, p. 4]

In response to Weatherall, Bryan Roberts objects to this dictum that 'isomorphic mathematical models in physics should be taken to have the same representational capacities'. Roberts thinks it would constitute 'a crippling limitation on mathematical modelling' since then 'the only legitimate mathematical representations are those that perfectly and completely describe the physical world' [8, p. 2]. Roberts's main point is that factors 'outside of a given mathematical formalism may distinguish between two descriptions, even when the formalism itself does not' [8, p. 5]. Applied to the hole argument:

> Let M denote the set of spacetime points in our universe. According to the manifold substantivalist considered by Earman and Norton (1987), this set is no abstract mathematical entity, but a concrete set in in [sic] the physical world. This would entail that there is a matter of fact about whether a star passes through that point in its lifetime, about what the value of the metric field is at that point, and about any number of other properties that are left undetermined by the hole transformation. ... Thus, in spite of Weatherall's dictum, the manifold substantivalist has no problem

> [using] these real-world facts to distinguish between two descriptions (M, g_{ab}) and $(M, \tilde{g}_{ab})$ that are related by an isometry. [8, p. 8]

For Roberts the isometric spacetimes are indeed equivalent *as Lorentzian manifolds,* but to claim they're the *same* is to prohibit distinct isomorphic things.

It is time to present my view on these matters. In the next section I make some distinctions about mathematical objects and modelling, in the following I propose a theory of symmetry. Later I return to evaluate the claims above.

2.2

The first distinction is between ways of counting mathematical objects; ways of quantifying over them. The philosophical commitments in mathematical interpretation that I'll presuppose are fairly minor; the method is read off mathematical practice and in any case quantifies mathematical objects in the 'logical sense' that Saunders (from Quine) used above.

Mathematicians are most interested in structure (whatever that is), and the first mode of quantifying I'll call *classificational,* for it is used to classify structures. In this sense there is *one* group of order 5 (namely, Z_5); there are *two* groups of order 6 ($Z_2 \times Z_3, Z_6$); there is *one* cardinal number between 2 and 4; there is *one* closed orientable surface with genus 5, and there is *one* smooth 4-manifold of the type of spacetime.

The *definitional* sense counts the ways a structure can be defined. I can't quantify them existentially, but at least Zermelo's three ($= \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$) and von Neumann's three ($= \{\{\{\emptyset\}\}\}$) are two different ordinal threes. See Benacerraf [9] for an argument from this ambiguity that numbers cannot *be* actual sets.

In the *concrete* sense every actual instantiation of a structure is distinct. Every set of 3 objects is a different 3; two physical spheres are two spheres. There is ambiguity in 'actual': we might like to include mathematical objects themselves when they act (at least psychologically) as the substance underlying *further* structure. Suppose I ask: how many subgroups of order 2 are in $Z_2 \times Z_2$? In the classifying sense there is only Z_2, but if we allow a concrete sense there are three. How many circles are in $\mathbb{R}^2$? If we consider $\mathbb{R}^2$ as a concrete set of things, ordered pairs of real numbers, then we can parametrize all concrete circles with two real numbers (center) and a positive real number (radius). So there are $\mathbb{R}^2 \times \mathbb{R}_{>0}$ circles. We need not assume '3' or '$\mathbb{R}$' are concrete *simpliciter,* but they have concrete roles in these contexts.

If we do allow 'mathematically concrete' things, as I think we should, we can make another nice distinction that will do a lot of work sorting the clutter in the Weatherall-Roberts dispute. I see two ways to model the world mathematically.

The first I call *abstract structure instantiation* (ASI) and the second *concrete object representation* (COR). In ASI the scientist asserts only that the physical world or something in the world has the mathematical structure of, say, a 2-sphere (manifold category); she asserts an object in the classifying sense instantiated in the world. In COR the scientist takes a concrete 2-sphere, say $\{x \in \mathbb{R}^3 : \|x\| = 1\}$, and asserts that it *represents* somehow something in the world. Or closer to home, we might say that spacetime has the structure of a smooth 4-manifold or Lorentzian 4-manifold (ASI), or we might rather imagine a specific 4-manifold, say as dwelling in a coordinate atlas, stitched together from chunks of $\mathbb{R}^4$ (Klein's program), and claim this represents spacetime.

What is the difference? In ASI we assert the structure, and only the structure, given under a type. We don't assume any particular definition of the structure; in fact we might say that a definition *amounts to* a canonical concrete (mathematical) instantiation. Take again a 2-sphere in the world according to ASI. On some definitions the 2-sphere is a primitive element without parts, and for example its symmetry is realised as a kind of primitive property. More typically it will have points as parts; but the sphere structure only distinguishes the points weakly (in Saunders's vocabulary). But perhaps the points have other properties, say an individuating field. None of these physical possibilities are precluded, but none asserted. An object in the classifying sense has symmetry characterized by its automorphism group (also quantified abstractly); the *minimally structured* instantiation would have a perfect symmetry of that type.

In the case of COR we presuppose a particular instantiation in the concrete 2-sphere. This approach gives some ontological wiggle room, in two ways. Firstly every concrete thing has ambiguous structure (fits under multiple types); just so, it isn't *a priori* clear what structure is had by 'the unit sphere in $\mathbb{R}^3$'. (Does it include the third digit in the decimal expansions of the second coordinate?) Secondly 'represents' is also ambiguous, perhaps for the same reason. We aren't told exactly *how* a thing represents: what aspects are mirrored?

A brief aside: why not include also COI and ASR? Quite simple: forms and not things are instantiated, and things not forms represent.

Let us not venture into the myriad philosophical problems provoked by my distinction. (Platonism, Putnam's Paradox, etc.) I think the distinction is intuitively clear, practical illustrations proliferate, and it helps mightily to clear up our mess. So let me next give my theory of symmetry.

2.3

I would say that whenever a symmetry is genuine the symmetry transformations are meaningless. Take a perfect circle. I would say it is meaningless to 'rotate' the circle. How could one grab on? We can break the symmetry by introducing coordinates, and then rotate the coordinatized circle. It is not the original circle we rotate, but one set of coordinates against another. We have not *given* coordinates to the circle, in the sense that each coordinate now refers to one point on the circle rather than another, for that is impossible. The coordinates only refer to themselves and we find the circle form instantiated within the coordinates.

Symmetry so conceived is a brute property of a thing. Symmetry comes in different flavours (classified by groups) and exists relative to a structural kind (in a category). Perhaps these kinds are idealizations; but no matter. The pure form has pure symmetry.

We have a standard technique to *augment* symmetry in concrete things. We pretend that a collection of unsymmetric things are really one; this is the method of equivalence classes and quotient structures. We can rotate the coordinatized circle and take 'whatever they have in common' (reify the equivalence class). The class better represents the symmetric form of the circle, though it's still concrete. But I think that if we can abstract anything at all, we can abstract the circle itself, and the abstract circle already has perfect symmetry. Then if we assert a circle in the world (classificational circle and ASI modeling) by default the circle-shaped thing also has this symmetry, and rotation is senseless. We can break the symmetry first in our mind-lab and assert the *coordinatized circle* (via COR); then we are inclined to think coordinates label things in reality; or at least taking this circle to mirror reality leaves open that possibility.

3

Let's return to Weatherall and the hole problem. I say that Weatherall blurs the line between classificational and concrete quantification of mathematical objects, and that between ASI and COR. Crossing the first line he holds 'the view that mathematical models of a physical theory are only defined up to isomorphism': he is willing to countenance a plurality of distinct and isomorphic models and is not willing that the distinction *mean* anything, since he only considers comparison maps in one category at a time. Crossing the second he thinks 'isomorphic mathematical models in physics should be taken to have the same representational capacities': he wants a model to carry only abstract structure but yet various distinct models to *represent* the same physical circumstance. I would say, only a concrete object of the mind can re-present a concrete object in the world.

Roberts isn't clear on how to quantify mathematical objects generally, but he takes for granted that objects in mathematical models come with additional (ambiguous) structure beyond the pure abstract form of the category in question. From this and his examples (cf. rotation, [8, p. 4]) I think it clear he embraces concrete mathematical objects and not abstract forms, missing the classificational sense. But his view is definitely one of instantiation ('Let M denote the set of spacetime points in our universe') not representation. The hole problem returns in full force as he perceives. But since his concrete spacetime will include the individuating structure of coordinates, I think he cannot escape.

How then do I answer the hole argument? The simple answer is that with the benefit of hindsight on Einstein's theory we should use ASI and assert the pure smooth manifold structure as the background spacetime on which the metric is a field (though I will be the first to grant the metric special status among fields). The manifold structure has symmetry characterized by the group Diff(M), and that symmetry for spacetime is *genuine*. This implies there are no symmetry *transformations* and coordinates *do not refer to things* for there is no way to refer to single points (if there even are points, and e.g., we needn't prefer points to embedded curves and surfaces). One must speak about the whole object, *or* make statements that respect the symmetry (which means: when working with the symmetry broken, applying symmetry transformations doesn't change their validity). 'Natural' arguments of pure mathematics follow this rule. In all this I'm in full agreement with Saunders:

> But in fact the points of space are only weakly discernible, so we cannot refer to any one point rather than another, and the difficulty does not arise. Evidently *insofar as we can view the parts of a highly symmetric entity, such as a homogeneous space, as objects in their own right* (as discernibles), without reference to anything else, it is essential - consistent with the [Principle of Sufficient Reason] - that they not be absolutely discernible from one another. [6, p. 15, my italics]

I differ in emphasizing symmetric wholes: one only loses data, the data of the specific *kind* of symmetry involved, by transposition to talk of parts as individuals. I think there is something quite unnatural about an object without internal nature, and atomic parts of ideal structures are such objects.

Symmetric spaces often model backgrounds to which other structures or objects ('furniture') are added. Doing so breaks the symmetry, in a simple sense. I do not mean one can refer to those same points of the old symmetric space by way of the new furniture, for again there is no sense to saying the furniture is spread one way or another over those points. But in the new picture *with* the individuating furniture field, I think it's quite acceptable to refer to points relative to the field. For now the points are absolutely discernible; but the points are parts of a different world, a world with furniture.

I will end with a remark on the possibility of a coherent position using COR, and a suggested explanation for the origin of the whole difficulty.

We could of course stipulate that a smooth manifold as given in a particular coordinate atlas represents spacetime. We *could* recover the right result by further stipulation that only the smooth structure, with attendant genuine symmetry, is to be transferred to the world, and not the coordinate labels. We get the same effect as in the other method. A more honest way to achieve this result is to quotient out the labels, by taking as concrete mathematical object the equivalence class of all atlased-manifolds with given smooth structure (a collection of concrete sets); this is like taking the equivalence class of all sets of 3 things to represent the cardinal 3. All these sets share the property of having three elements so that will be a property of the class; similarly the concrete manifolds share all smooth manifold properties of interest. Alternatively one could take the orbit of the first atlased-manifold under the symmetry transformations. Yet another option is to adopt a synthetic geometry and simply postulate the symmetric space as a primitive. In the end one has a concrete object with genuine symmetry, and that would stand to represent spacetime.

I think this gives a clue about the deepest roots of our dilemma. The distinction between ASI and COR seems general and robust, but I think it's telling that the main application coincides with a distinction between synthetic geometry, where shapes are primitives, and analytic, where everything is constructed from real numbers. In a logical space for abstract geometry where the smooth 4-manifold of spacetime is a primitive, it would be hard to see much difference between these methods of modeling. In any case one might take the history of physics to be a story of identifying 'physical quantities', or numbers, and relating them with equations. This came with Descartes to include space itself, its quantity enumerated by coordinates. But this is already to choose a concrete model opposed to covariance, more generally, opposed to a synthetic theory of symmetric shapes and spaces; and as soon as the generically non-symmetric metric is freed from the smooth background, we will perceive a redundancy corresponding to the supposed absolute values of those quantities of place. We should, rather, allow ourselves mathematical and physical objects with genuine symmetry to describe homogeneous background spaces, and the hole transformations will fall, even in the realm of pure mathematics, from real motions that allegedly do nothing, to real nothings; we will accept Leibniz Equivalence even for the models themselves, and spacetime will be a highly symmetric substance.

References

[1] John Earman and John Norton. What Price Spacetime Substantivalism: The Hole Story. *British Journal for the Philosophy of Science*, 38:515–525, December 1987.

[2] Tim Maudlin. The Essence of Space-Time. *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association*, 1988:82–91, 1988.

[3] Jeremy Butterfield. The Hole Truth. *The British Journal for the Philosophy of Science*, 40(1):1–28, 1989.

[4] Carolyn Brighouse. Spacetime and Holes. *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association*, 1994:117–125, 1994.

[5] Carl Hoefer. The Metaphysics of Space-Time Substantivalism. *The Journal of Philosophy*, 93(1):5–27, 1996.

[6] Simon Saunders. Physics and Leibniz's principles. In Katherine Brading and Elena Castellani, editors, *Symmetries in physics: philosophical reflections [electronic resource]*. Cambridge University Press, Cambridge, UK; New York, 2003.

[7] James Owen Weatherall. Regarding the "Hole Argument". *arXiv ID: 1412.0303*, 2015.

[8] Bryan Roberts. Disregarding the 'Hole Argument'. *arXive ID: 1412.5289*, March 2015.

[9] Paul Benacerraf. What Numbers Could not Be. *The Philosophical Review*, 74(1):47–73, 1965.